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RESEARCH MEMORANDUM

Improving the Quality of Numeracy Skills: Progressing from Level 2 to Level 4 on the PIAAC Cycle 2 Proficiency

An ETS Return on Investment Study: Phase 1

AUTHORS

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Improving the Quality of Numeracy Skills: Progressing From Level 2 to Level 4 on the PIAAC Cycle 2 Proficiency Scale: An ETS Return on Investment Study, Phase 1

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Preface

This research memorandum highlights the collaborative efforts of the numeracy panel for the ETS Return on Investment (ROI) Study, Phase 1, comprising David (Dave) Tout, Kees Hoogland, and Javier Díez-Palomar, in developing their research memorandum, *Improving the Quality of Numeracy Skills: Progressing from Level 2 to Level 4 on the PIAAC Cycle 2 Proficiency Scale*. This memorandum captures the expert opinions of the panel sessions. It is provided as supporting documentation for the policy report, *Level Up: Raising the Skills of Adults in the United States and Other Countries* (Irwin Kirsch, Mary Louise Lennon, and Anita Sands, with Jean-François Rouet, Anne Britt, Tobias Richter, Dave Tout, Kees Hoogland, and Javier Diez-Palomar). A second ROI study, which also supports the *Level Up* policy report, addresses literacy: *Literacy Skills at and Around Level 2 of the PIAAC Cycle 2 Proficiency Scale* (Jean-François Rouet, Anne Britt, and Tobias Richter).

The ROI project, funded by ETS and led by Irwin Kirsch (retired) and Anita Sands of the ETS Research Institute, presents findings and recommendations based on the panelists' expertise and thorough analysis. However, these findings and recommendations should be interpreted with caution as they have not undergone formal peer review. This memorandum is intended for informational purposes only. ETS does not endorse or assume responsibility for any conclusions or recommendations made by the panelists. The views expressed are solely those of the authors and do not necessarily represent the official policy or position of ETS. Any errors or omissions are the responsibility of the authors.

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Paired Reports

Policy Report: *Level Up: Raising the Skills of Adults in the United States and Other Countries* by Irwin Kirsch, Mary Louise Lennon, and Anita Sands, with Jean-François Rouet, Anne Britt, Tobias

Richter, Dave Tout, Kees Hoogland, and Javier Diez-Palomar

https://www.ets.org/Media/Research/pdf/RR-25-04.pdf

Research Memorandum: *Literacy Skills at and Around Level 2 of the PIAAC Cycle 2 Proficiency Scale* by Jean-François Rouet, Anne Britt, and Tobias Richter

https://www.ets.org/Media/Research/pdf/RM-25-01.pdf

Research Memorandum: *Improving the Quality of Numeracy Skills: Progressing from Level 2 to Level 4 on the PIAAC Cycle 2 Proficiency Scale: An ETS Return on Investment Study, Phase 1* by Dave Tout, Kees Hoogland, Javier Díez-Palomar

https://www.ets.org/Media/Research/pdf/RM-25-02.pdf

Introduction

Societies are changing rapidly due to digitalization, technologization, and artificial intelligence. It has become increasingly clear that foundational education (K–12) is not sufficient to equip individuals with the necessary skills to cope with the demands of 21st century workforces and society.

Increasingly, research is showing that life and work in the 21st century requires higher levels of mathematics and numeracy of its citizens. The skills and knowledge that are needed to succeed in work, life, and citizenship have significantly changed in the 21st century, often driven by technological advances and an ever increasing use of numerical and quantitative information and data. The transforming nature of the workforce associated with “industry 4.0,” the “gig economy,” and the impacts of generative AI intensify the demands for science, technology, engineering and mathematics (STEM) skills (e.g., see Australian Association of Mathematics Teachers & Australian Industry Group, 2014; Binkley et al., 2012; Foundations for Young Australians, 2017; Gravemeijer et al., 2017; Griffin et al., 2012; Hoyles et al., 2010; National Council of Teachers of Mathematics [NCTM], 2017; Partnership for 21st Century Skills [P21], 2016; Pellegrino & Hilton, 2012). Increased reliance on technology to automate industrial processes, and on big data mining to make decisions in industry and society, elevates the importance of critical thinking in numeracy and mathematical sense.

In their 2017 review of mathematics education for the 21st century, NCTM argued that mathematics is at the heart of most innovations in the information economy. They saw mathematical and statistical literacy as needed more than ever to filter, understand, and act on the enormous amount of data and information that we encounter every day. Many companies and workplaces are now seeking workers who are able to read and understand statistical and/or mathematical information to check, review, and proactively act on information and process-oriented chains, prompts, procedures, and so on. Numeracy is also needed in other everyday contexts, such as exercising the right to participate in democracy, maintaining a healthy lifestyle, managing finances, and making sensible life decisions. Consequently, numeracy extends beyond merely possessing a certain level of mathematical understanding; it is a multifaceted competence that encompasses the proficient application of this knowledge within various contexts (e.g., see Gal et al., 2020; OECD, 2021; Tout et al., 2017).

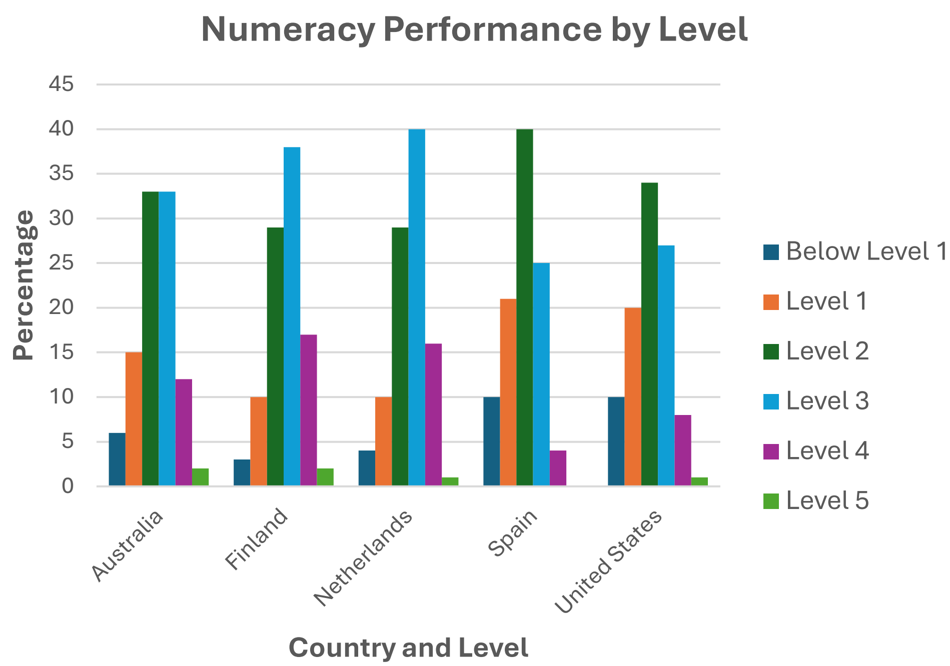
Return on Investment From Improving Numeracy Skills

Based on the three previous cycles of international assessments of adult literacy and numeracy skills conducted since 1994 (IALS, ALL, and PIAAC), research indicates that on average people with higher literacy and numeracy skills are significantly more likely to be employed, to participate in their community, to report better health, and to engage in further education and training. They also earn more on average. Findings from PIAAC Cycle 1 have reinforced this research (see e.g., Hanushek et al., 2015; Lane & Conlon, 2016; OECD 2013, 2016, 2019; Tout, 2019; Vignoles & Cherry, 2020). There is also some evidence suggesting that compared to equivalent literacy skills, numeracy skills are associated with higher earnings (at each level of education; Lane & Conlon, 2016, p. 21)—a relationship similarly observed within other variables—such as employment and health (Tout, 2019).

An analysis of data from the first cycle of PIAAC highlights the distribution of numeracy proficiency levels among adults. Figure 1 illustrates numeracy performance across five countries from PIAAC Cycle 1, while Table 1 consolidates these findings, summarizing the proportion of adults performing at Level 2 or below and Level 3 or below. It is widely accepted that a minimum Level 3 numeracy proficiency is often necessary for adults to effectively meet the growing demands of daily life and work. This distribution provides insight into where targeted investments in skill advancement between levels may yield the greatest impact, specifically improvements geared for the substantial segments of the population at Level 2 or below. Improving numeracy skills in this group could enhance individual opportunities, while also delivering broader societal and economic benefits (Hanushek et al., 2015).

While these data are not longitudinal, the observed decline in numeracy skills with age (see Figure 2) suggests that investing in workforce development to support transitions from Level 2 to Level 3 may help mitigate these declines, fostering sustained proficiency and adaptability.

Figure 1. Distribution of PIAAC Numeracy Levels Across Select Countries



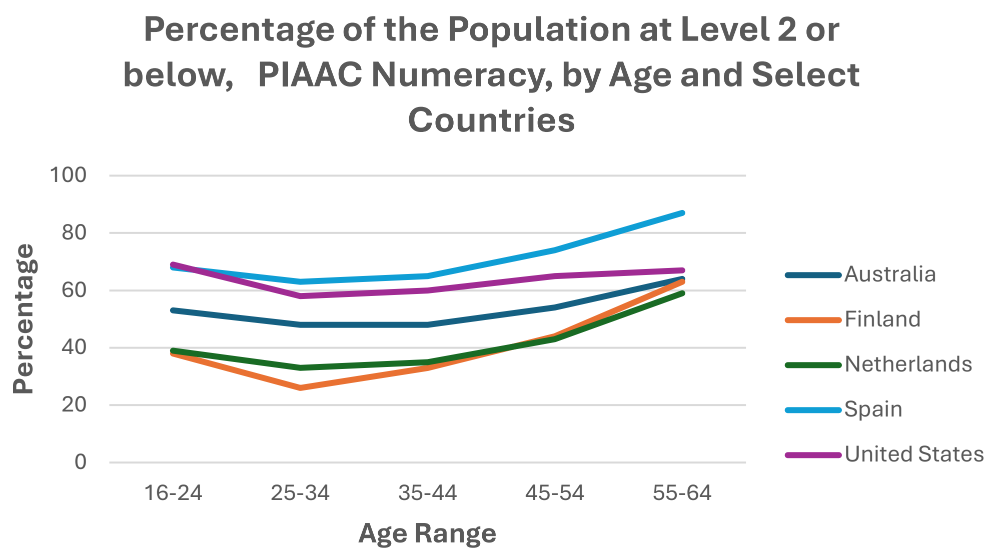
Source: PIAAC Cycle 1 microdata, 2011-2017 Organisation for Economic Co-operation and Development (OECD).

Table 1. Cumulative Percentage Distribution for Levels 2 or Below and 3 or Below in PIAAC Numeracy, Select Countries

Country	Percent at Level 2 or below	Percent at Level 3 or below
Australia	54	87
Finland	42	80
Netherlands	43	83
Spain	71	96
United States	64	91

Source: PIAAC Cycle 1 microdata, 2011-2017 Organisation for Economic Co-operation and Development (OECD).

Figure 2. Percentage at Level 2 or Below in PIAAC Numeracy by Age



Source: PIAAC Cycle 1 microdata, 2011-2017 Organisation for Economic Co-operation and Development (OECD).

Potential Skill Improvements From of an Integrated Literacy and Numeracy Intervention Program

The expected skill improvements for adults participating in a work-based literacy and numeracy education intervention program is difficult to predict for a number of reasons and could vary widely due to factors such as the type and length of the program, the support structures in place, and each individual's background and motivation. Drawing on our experience and knowledge of the research, many targeted work-based education programs demonstrate improvements in adult skills. However, studies specifically evaluating the effectiveness of literacy and numeracy intervention programs are less prevalent and conclusive (e.g., Clayton, 2019). Generally, work-based programs that offer clear career pathways, relevant skills training, and strong support services tend to see better completion rates and gains for participating adults. We believe certain factors would contribute to greater effectiveness of adult skills programs, including ensuring that programs closely align with current job demands; access to mentoring, counselling, and additional resources; flexible schedules; and opportunities to study and apply skills. Targeted literacy and numeracy interventions should be built with these factors in mind. The key purpose would be to make the training relevant to the circumstances of the industry or business where the workers are located and address the required literacy and numeracy knowledge and skills required to perform workplace tasks.

The benefits of such programs would not be restricted to improvements in participants' literacy and numeracy skills and their ability to take on more demanding tasks, but also benefit the workplace from an economic and productivity perspective. As noted in Australian research by Brown et al. (2015):

> We wanted to establish a business case for employers to invest in this type of training and so the Building Employer Commitment to Workplace Literacy and Numeracy Programs project was developed. The particular focus of this project was to establish the return on investment for employers who participate in programs. . . . The results from the research are very impressive. For those companies that were able to generate a return on investment calculation the results were all positive in the range of 102 to 163 per cent. These results were achieved across a number of different States and industries such as manufacturing, utilities, construction, and aged care. These positive results strengthen the business case for employers to invest. In addition to other reasons for implementing workplace reform in this area, it now also makes good economic sense. We hope that all employers will heed these results and engage in foundation skills training for their workforce. (Brown et al., 2015, p. 4)

However, we need to remember that there are a range of issues that prevent adults from perseverance and completion of such a training program. Adults drop out of adult education or training programs for various understandable reasons, including the need to balance education with work, family responsibilities, and other commitments that create a challenge for adults to fit classes or training sessions into their busy schedules. These obstacles include, but are certainly not limited to the following:

- Lacking strong support system from family, friends, coworkers, and employers that can make it difficult to stay motivated and committed to completing a program

- Health issues, family emergencies, or personal crises that can disrupt an adult's ability to focus on their education or training
- Without a clear understanding of how the program will benefit their career or personal growth, adults might lose motivation and drop out.
- If the program does not meet the adult's expectations or needs in terms of content, instruction, or resources, they might decide it is not worth continuing.

As mentioned earlier, addressing these challenges often requires tailored support and flexible program designs that consider the specific needs and circumstances of adult learners.

What percentage of adults are likely to improve their literacy and numeracy performance if they participate in work-based adult literacy and numeracy intervention program?

In addition to the generic delivery of many literacy and numeracy programs, the prevailing issue with much existing research about the success of adult literacy or numeracy programs stems from the fact that most programs target the lower performing adult learners. In such research, the results and outcomes are highly variable.

With the proposed approach to interventions outlined in this paper, the aim is to specifically target learners in the middle brackets of existing literacy and numeracy performance, as illustrated in Figure 1 and Table 1 (i.e., Levels 2 and 3 of PIAAC). In targeted programs that address some of the factors required to support effective work-based adult education, the rate of skill advancement among adults is anticipated to be significant. In addition, the potential for improvement is likely to increase when progressing from a lower to a higher level, as a higher starting point suggests greater capacity and a stronger foundation for more rapid and efficient advancement. Based on the authors' experiences and expertise, it is reasonable to expect that gains may fall within a range of 60% to 80%.

As a proxy for this type of targeted program, one of the authors of this paper analyzed the success rates of the Victorian Certificates of Applied Learning (VCAL) program, which is recognized within the Australian Qualifications Framework (AFQ), the national policy that governs the recognition of educational qualifications in Australia (VCAA; 2021). This program targets youth in their final years of schooling who did not intend to follow a traditional pre-

University academic focused pathway. VCAL was tailored for students between ages 16 and 18 seeking practical, hands-on learning that prepares them for employment or further education. In addition, VCAL was structured around applied learning principles, with an emphasis on employability and vocational skills across real-life contexts, including workplaces, community projects, and industry settings (Victorian Curriculum and Assessment Authority [VCAA], 2021). The VCAL framework is divided into four key educational strands: literacy and numeracy skills, industry-specific skills, work-related skills, and personal development skills that are then spread across three levels—foundation, intermediate, and senior. Each strand and level catering to different skill levels and progressively building students' skills. Assessments in the VCAL program are central to measuring students' mastery of learning outcomes in applied, real-world skills and focused on gathering diverse evidence of achievement—such as through direct observations, written work, oral presentations, product creation, project implementation, and for some, summative assessments—to ensure students meet specific, practical learning goals (VCAA, n.d.). This course and its cohort of nonacademic learners share sufficient similarities with aspects of the target groups discussed in this paper to warrant consideration of the proposed intervention.

While the proficiency levels for the VCAL course are not identical to those reported in PIAAC, two of the levels (intermediate and senior) were judged to be reasonable proxies for PIAAC Level 2 and 3.[1] Therefore, an analysis of success rates in completing those two VCAL levels could provide insight into the potential success rate of the proposed strategy for developing targeted interventions. Using the second and third levels as proxies for Levels 2 and 3 on the PIAAC scale as is the focus of this paper and its proposed intervention, it is possible to analyze the success rates of students in completing these two levels of VCAL.

Looking at the published data on completion rates from 2019 through to 2022 for the Units in each semester, the success rates for intermediate numeracy units ranged from 69.3% up to 76.9% over the year. This compares with a success rate ranging from 79.2% up to 88.3% for the senior level. These results would seem to support that the expected success rates of proposed intervention strategies herein of between 60% and 80% are not unreasonable and would most likely be at the higher end when moving adults from Level 2 to Level 3 and even higher when progressing from Level 3 to Level 4, compared to moving adults from Level 1 to Level 2 (VCAA, n.d.).

About This Working Paper

This working paper describes the work of the numeracy experts who conducted analyses of the PIAAC data and the assessment tasks themselves with the goal of clearly defining the knowledge and skills associated with various levels of proficiency as described in PIAAC numeracy. The focus of the work was to describe and explain the skills that reflect performance at key proficiency levels along the numeracy scale. The key purpose was to inform the development of strategic interventions to improve adult literacy and numeracy skills.

The underlying premise of the project was that developing an understanding of the underlying progression of skills that are needed to successfully perform on the kinds of tasks that people need to do at work, at home, and in their communities could form an empirically based foundation for linking the assessment frameworks with instruction that could be designed to help adults improve their skills.

The three numeracy experts working on this project were no strangers to PIAAC. They all are recognized for their experience and expertise in numeracy and mathematics education for adults, and each of them was a member of the numeracy expert group who developed the PIAAC framework document, oversaw the development of the assessment tasks, and led the interpretation of the survey results.

This working paper describes the skills that support numeracy proficiency as learners progress through the described levels of performance on the PIAAC Cycle 2 scale, with a focus on progress from Level 1 through to Level 4. The data collected through Cycle 2 of PIAAC provide a good basis for analyzing which numeracy and mathematical knowledge and skills should be improved to support adults transitioning from level to level. This paper considers what educational interventions would be needed to improve the quality and outcomes of numerate behavior of an individual when progressing from Level 2 to Level 3 based on the Cycle 2 data, while also considering transitioning from Level 1 to Level 2, and from Level 3 to Level 4.

This document is based on the descriptions, construct, and assessment outcomes for numeracy in Cycle 2 of PIAAC, and especially on the document *The Assessment Frameworks for*

Cycle 2 of the Programme for the International Assessment of Adult Competencies (OECD, 2021). The framework background and structure are elaborated further in Appendix A. However, it also needs to be acknowledged that a range of other aspects will impact and drive the effectiveness of any such intervention. The enabling factors presented in Appendix A describe this more fully.

We began our analysis and research looking at the overarching framework and its different dimensions that describe the range of skills needed to perform numeracy tasks at any level of proficiency. We also considered the existing ALL and PIAAC Cycle 1 Numeracy Task Complexity schema (Tout et al., 2020). The next step was to identify which of these dimensions, as described in the PIAAC Cycle 2 framework, would appear to drive proficiency in numeracy. Next, we undertook an analysis and comparison of all the PIAAC Cycle 2 items against each of these dimensions, progressing up the levels from Level 1 through Level 4. We described what we found and looked for the factors that changed moving from one level to another in relation to the PIAAC Cycle 2 construct dimensions. Fuller descriptions of what we found are located in Appendix C. These findings are summarized below in the Differences Between Proficiency Levels in Numeracy section.

The final two tasks for the working group involved focusing on and generalizing about any common features or key aspects of the dimensions that we believe are the key indicators of complexity, progressing up the levels of numeracy proficiency, and following that to develop and describe what interventions might work best to support adults to move from one level of numeracy performance to the next higher level of performance.

Key Features of the PIAAC Cycle 2 Numeracy Framework

PIAAC is an assessment that measures the proficiency of adults in three main domains: literacy, numeracy, and adaptive problem-solving (APS). Table 2 provides a schematic representation of the different dimensions and processes associated with each domain—literacy, numeracy, and problem-solving in technology-rich environments. The features of the numeracy assessment framework for PIAAC Cycle 2 are elaborated further in Appendix A.

Table 2. Main Features of the Assessment Frameworks for PIAAC Cycle 2

Feature	Literacy	Numeracy	Adaptive Problem Solving
Definition	Literacy is accessing, understanding, evaluating and reflecting on written texts in order to achieve one's goals, to develop one's knowledge and potential and to participate in society.	Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.	Adaptive problem solving involves the capacity to achieve one's goals in a dynamic situation, in which a method for solution is not immediately available. It requires engaging in cognitive and metacognitive processes to define the problem, search for information, and apply a solution in a variety of information environments and contexts.
Cognitive processes	• Accessing text • Understanding • Evaluating	• Access and assess situations mathematically • Act on and use mathematics • Evaluate, critically reflect, make judgements	• Definition • Searching • Application
Content	Texts characterized by their: • Type (description, narration, exposition, argumentation, instruction, transaction) • Format (continuous, non- continuous, mixed) • Organisation (the amount of information and the density of content representation and access devices) • Source (single vs. multiple texts)	**Mathematical content, information and ideas** • Quantity and number • Space and shape • Change and relationships • Data and chance **Mathematical Representations** • Text or symbols • Images of physical objects • Structured information • Dynamic applications	Task dimensions • Problem configuration • Dynamics of the situation • Features of the environment • Information environment
Contexts	• Work and occupation • Personal • Social and civic	• Personal • Work • Societal/community	• Personal • Work • Social/community

Source: OECD (2021), page 19.

In this paper, we focus on the domain of numeracy. From these features, three numeracy dimensions are seen as especially relevant for this paper. Those are the dimensions of

- cognitive processes,
- mathematical content, and
- representations.

The research team believes that these three dimensions are the main dimensions that will help in defining the level of performance of individuals, and they will be recognizable in the proposed interventions at the end of this paper. A fourth dimension of *context*, whilst important in relation to teaching and learning practices, is not considered to be a factor that drives difficulty of a numeracy question or task per se. Context as described in the framework (work and occupation, personal, and social and civic) is used to describe the setting or situation in which the numeracy questions are situated and allows a spread of such contexts to be utilized across the assessment item pool. Context is seen to be important only in general terms, in relation to whether the situation is very familiar and hence common, to most respondents at the lower skill levels, whereas at higher levels, the context is expected to be less familiar or rarely encountered in an adult's life. The specification of the context against one of the PIAAC context categories will not drive task complexity in its own right.

In PIAAC numeracy, the dimension named cognitive processes pertains to individuals' mental activities, processes, and tasks when attempting to resolve questions or problems across various contexts, including real-life, authentic, or educational settings. The mathematical content dimension encompasses the range of mathematical knowledge and skills that underpin applying these cognitive processes. The representation dimension pertains to how the quantitative and mathematical information is embedded in real-world situations and contexts, whether in words and text, diagrammatically or graphically, or dynamically. Mathematics, per se, does not exist in the real world by itself in its own isolated abstract form. These three dimensions are elaborated in detail in Appendix A.

It should be noted that in the real-world of solving authentic numeracy tasks, these four dimensions are not separate skills; they are integrated and intertwined within the task at hand

and adults will bring to bear a wide range of knowledge and skills, and also behaviors and attitudes, in order to respond to the problem. The purpose of the PIAAC construct and these dimensions are to attempt to separately describe the components of an assessment construct that can be used to assess numeracy proficiency across a very broad adult population. This working paper utilizes that PIAAC framework and its construct to review and analyze the actual proficiencies of adults across the empirical PIAAC proficiency spectrum that resulted from PIAAC Cycle 2.

Differences Between Proficiency Levels in Numeracy

Introduction and General Remarks

The distinctions between proficiency levels, which describe what adults can accomplish and the extent of their knowledge across the three domains (literacy, numeracy, and problem-solving), stem from varying degrees of complexity in the cognitive processes involved, as well as the difficulty of the content within the tasks, situations, or problems encountered.

Across all three domains, common factors influence the difficulty of tasks. For example, the task statement plays a crucial role; sometimes, instructions are explicit, while in other instances, they are implicit. In authentic situations, these domains may be interwoven to varying degrees, requiring adults to evaluate how to critically approach and solve the task. In straightforward contexts, instructions might be clear and questions direct, without distracting elements. However, in more complex contexts, information may be less apparent, embedded within the situation, demanding that adults effectively read, interpret, understand, and judiciously apply their knowledge and skills to identify an appropriate solution to the given task or problem.

For instance, in relation to numeracy specifically, some tasks may be straightforward, featuring simple texts or visual representations of objects, clear questions from a single source, and basic mathematical content such as whole numbers and requiring only a single step for resolution (such as adding), with explicit questions and all necessary information provided in the text without distractions. Conversely, we may encounter situations where the mathematical content is complex, with more sophisticated forms of representation (such as in a mathematical formula or multivariable chart or graph), and where the steps required to solve the task are not clear or explicit but rather embedded within the context. In such cases, one must analyze, comprehend, and determine the appropriate question, mathematical information, and potential pathways for resolving the situation.

Between these extremes, there exists a wide range of situations and levels of challenge and complexity. In PIAAC Cycle 2, a scale is employed to structure and differentiate these various proficiency levels.

The PIAAC Scale and Proficiency Levels

In each of the three domains assessed, proficiency is considered a continuum of ability involving the mastery of information-processing tasks of increasing complexity. The results are represented on a 500-point scale. At each point on the scale, an individual with a proficiency score of that particular value has a 67% chance of successfully completing test items located at that point.

To help interpret the results, the reporting scales have been divided into proficiency levels defined by particular score-point ranges. Six proficiency levels are defined for literacy and numeracy (below Level 1 and Levels 1 through 5), with boundary scores of 175, 225, 275, 325, and 375.

The descriptors of these levels in the assessment framework provide a summary of the characteristics of the types of tasks that can be successfully completed by adults with proficiency scores in a particular range. In other words, they offer a summary of what adults with particular proficiency scores in a particular skill domain can do. Except for the lowest level (below Level 1), tasks located at a particular level can be successfully completed approximately 50% of the time by a person with a proficiency score at the bottom of the range defining the level.

For the purpose of this paper, we used the analysis of the PIAAC Cycle 2 assessment outcomes at the item level to elaborate these proficiency descriptions to get an even more detailed understanding of what adults can do at each of the described levels based on the data gathered in the PIAAC cycle 2 survey. Using these detailed descriptions and the analysis, we developed our suggestions for the educational interventions that aim to progress adults from one level to a higher level.

The ALLS and PIAAC Numeracy Complexity Schema

Previous research indicates that the factors influencing the complexity of answering a numeracy task do not only encompass the mathematical elements of tasks, but also other considerations as described in the PIAAC Numeracy Task Complexity schema (Tout et al., 2020). A situation's complexity is not solely determined by the mathematical content the task involves; for example, checking change when making a purchase is far less complex than selecting the most suitable mortgage plan or calculating the number of medicine bottles needed for a month-long holiday in the mountains, where 350 mg of medication must be consumed daily. Other aspects also contribute to complexity, such as the difficulty in reading and interpreting any written materials, the type of task, whether the questions posed are direct, the presence or absence of distracting information, and whether multiple sources of information are involved, necessitating a decision on which data are relevant.

Whilst the numeracy complexity schema helps interpret and understand some of the difficulties faced when answering a numeracy task, the existing PIAAC complexity schema was developed to categorize tasks and not to categorize the numerate behavior of adults. Certainly, the complexity of tasks can be extrapolated to assist in understanding the complexity of situations in real life that adults have to cope with, but in relation to developing appropriate instructional strategies for helping adults to improve their numerate behavior and competence, a range of other factors are critical.

However, as the factors in the PIAAC Numeracy Task Complexity schema have been utilized in the review of PIAAC Cycle 2 items and in this analysis, we summarize below the factors that were developed in this schema that were found to contribute to item complexity. Further detail and explanation of each of these factors is included in Appendix B with the full schema available publicly (Tout et al., 2020).

Table 3. PIAAC Task Complexity Factors—Overview

Aspects	Category	Range
Textual aspects	1. Type of match/problem transparency	Obvious/explicit to embedded/hidden
	2. Plausibility of distractors	No distractors to several distractors
Mathematical aspects	3. Complexity of Mathematical information/data	Concrete/simple to abstract/complex
	4. Type of operation/skill	Simple to complex
	5. Expected number of operations/processes	One to many

Problem transparency refers to how clearly the mathematical information and tasks are defined, encompassing elements such as the clarity with which procedures are outlined and the explicitness of the stated values. Type of match pertains to the process a respondent must undertake to connect the required action in the question to the relevant information in the task or text. This can range from a straightforward action, such as locating or matching information, to more complex actions requiring multiple searches through the provided data. This measure of complexity in a numeracy task also accounts for the extent to which mathematical information is embedded within the text. It also includes the existence of plausible distractors that may increase the complexity of the task.

Mathematical information or data complexity pertains to concrete objects (such items to be counted), simple whole numbers, or basic shapes and graphs. This information tends to be more familiar at lower skill levels, while at higher levels, it may become less familiar. Situations become increasingly challenging when they involve more abstract or complex information, such as very large or very small numbers, unfamiliar decimals or percentages, information related to rates, or dense visual data, as found in diagrams or complex tables.

Additionally, the type of operation or skill required is a significant factor. Sometimes, tasks demand basic operations such as addition, subtraction, division, or multiplication. However, other tasks may necessitate more advanced operations, such as powers, roots, first- or second-degree equations with one or more unknowns, the application of percentages, or the calculation of simple or compound interest, and so on. The nature of the operation adds to the complexity of the situation or task the adult is confronted with.

Lastly, the schema also considers the number of operations required to solve a task as a complexity measure: the greater the number of operations (and the more varied they are), the higher the task complexity.

Understanding these factors and their interactions to make numeracy questions more complex or not were also utilized in the analysis of the differences between the PIAAC proficiency levels in numeracy.

The Analysis Against the Three Key PIAAC Numeracy Dimensions

As described in the numeracy framework for PIAAC (OECD, 2021), numeracy is a complex, multifaceted construct comprised of multiple dimensions or components (e.g., see Ginsburg et al., 2006; Hoogland & Díez-Palomar, 2022).

> The cognitive and affective component of numeracy includes the processes that enable an individual to solve problems, and thereby links the context and the content (Ginsburg et al., 2006, p. 3)

Whilst an assessment such as PIAAC focuses on assessing the numeracy knowledge and skills of the respondents to the survey and assessment, it elaborates on the enabling processes including affective issues (see Appendix A) and incorporates aspects of the affective side of numeracy competence through the background questionnaire. The affective dimension is critical in the teaching, and learning of numeracy with adults will need to be considered in relation to any intervention strategies that are to be implemented.

In the following sections, we describe what we found in our analysis against the three key dimensions of the PIAAC numeracy framework and assessment that we believe drive the levels of difficulty or complexity across the PIAAC levels of proficiency, namely

- cognitive processes,
- mathematical content, and
- representations.

Cognitive Processes

The PIAAC Cycle 1 framework described three different cognitive processes that adults use to solve a numeracy problem under three broad categories: (a) identify, locate, or access; (b) act upon or use; and (c) interpret, evaluate/analyze, and communicate. In PIAAC Cycle 2, the cognitive domain has been reconceptualized and further elaborated into three components: (a) access and assess situations mathematically, (b) act and use mathematics, and (c) evaluate, critically reflect and make judgments. This is described further in Appendix A.

In PISA, the version of cognitive processes included a set of three similar, parallel processes: (a) formulating situations mathematically; (b) employing mathematical concepts, facts, procedures, and reasoning; and (c) interpreting, applying, and evaluating mathematical outcomes. The cognitive dimension of the PIAAC construct pertains to the mental processes through which we access, engage with, respond to, and evaluate our responses to situations involving embedded mathematics. This dimension is intimately connected to both the mathematical content and its representation. One of the main works on cognitive domains in mathematics, which has had a significant impact on the field of mathematics, was provided by Niss (2003) and Niss & Højgaard (2019). Niss used the concept of mathematical competence, which he defined as the ability to comprehend, assess, apply, and use mathematics across diverse contexts. According to Niss, mathematical competence encompasses eight specific competencies: (a) thinking mathematically, (b) posing and solving mathematical problems, (c) modelling mathematically, (d) reasoning mathematically, (e) representing mathematical entities, (f) handling mathematical symbols and formalisms, (g) communicating in, with and about mathematics, and (h) making use of aids and tools. The first four were grouped under the category *Posing and answering questions in and by means of mathematics*, whilst the second four were under the category *Handling the language, constructs, and tools of mathematics*. These categories were later used and redefined as the underpinning of fundamental capabilities in PISA. These categories and their underpinning meanings and descriptions were useful reference for the numeracy experts to also refer to in their work analyzing the PIAAC cognitive processes dimension, and the other dimensions as well.

PIAAC Cognitive Processes Dimension

The following key factors and related questions address the complexity of each of the stages of the PIAAC numeracy cognitive processes dimension (See Appendix A for a full elaboration of numeracy for PIAAC Cycle 2's three cognitive processes).

Access and Assess Situations Mathematically

- How was mathematics represented through words and language? Symbols, diagrams, pictures, graphs, and charts? Informal, formal, or complex mathematics representations and mathematical information?
- What is the degree of transformation required in a real-world situation? Implicit/explicit/obvious? Is the question unambiguous?
- What literacy and comprehension skills are required?
- How well does the mathematical solution match the context of the original real-world situation?

Act On and Use Mathematics

- How difficult are the mathematical concepts, facts, processes, and procedures?
- What is the level of mathematical reasoning, arguing, manipulating, and computing required?
- The number of steps/processes required

Evaluate, Critically Reflect, and Make Judgments

- How complex is it to evaluate, reflect, justify, explain, and connect the mathematical outcomes with the real-world context?
- How complex is it to connect the mathematical evidence to the real-world problem?

These three factors influence complexity both individually and in combination. In some instances, an adult's situation may involve explicit mathematical content that requires no interpretation to determine the necessary operation for arriving at an appropriate solution. However, in other cases, mathematics may be highly embedded within the situation, requiring the adult to infer which calculation or mathematical reasoning will lead to an adequate solution and sound decision-making.

This complexity may also be compounded by the difficulty of the concepts, facts, processes, and procedures embedded within the situation. As a result, the level of reflection required from the adult can vary in complexity, depending on these contributing factors. The overall complexity of the task—encompassing the ability to access and assess situations mathematically, act on and use mathematics, and evaluate, critically reflect, and make judgements—depends on the combined complexity of all these components.

Based on the detailed analysis that is documented in Appendix C, the following sections summarize the aspects related to transitioning from a lower to a higher level of complexity on the PIAAC scale in relation to the cognitive processes.

The Development From Level 1 to Level 2 in Cognitive Processes

- At Level 1, adults can access, act on, and use mathematical information located in slightly more complex[2] representations set in authentic contexts where mathematical content is explicit. For Level 2, their access to, acting on, and use of mathematical information also includes more formal terminology and distractors.
- At Level 1, adults can devise simple strategies using one or two steps to determine the solution, whereas at Level 2, they can carry out multistep mathematical processes.
- At Level 1, adults are not expected to use mathematical reasoning to review or evaluate the validity of statements, whereas, for Level 2, we expect them to be able to reflect on and use mathematical reasoning when evaluating the validity of a statement.

Thus, when adults are transitioning between levels 1 and 2, at Level 1, adults can access, act on, and use mathematical information located in authentic contexts where mathematics is explicit, using one or two steps to solve a problem. At Level 2, adults can also solve tasks where the information includes more formal terminology and some distractors, using a multistep process to solve the problem.

The Development From Level 2 to Level 3 in Cognitive Processes

- At Level 2, adults can access, act on, and use information and evaluate simple claims using data for tasks set in a variety of authentic contexts. For Level 3, there is an additional focus on reflecting and evaluating more complex situations, including authentic mathematical contexts.
- At Level 2, adults are intended to be able to interpret and use information. At Level 3, people are required to make judgments about how to use the given information when developing a solution to a problem.
- At Level 2, the information is presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, or linear scales). For Level 3 the mathematical information may be less explicit, embedded in contexts that are not always commonplace, and use representations and terminology that are more formal and involve greater complexity.
- At Level 2, adults can carry out multistep mathematical processes. For Level 3, the mathematical processes require applying two or more steps where multiple conditions need to be satisfied. Tasks may also require using, integrating, or manipulating multiple data sources to undertake the mathematical analyses necessary for the specific task.
- At Level 2, adults can use mathematical reasoning when reviewing and evaluating the validity of statements and percentages or perform routine algorithms. For Level 3, adults reflect on and use mathematical reasoning when reviewing and evaluating the validity of conclusions drawn from data, including a limited set of related conditions.

Thus, when adults are transitioning between Level 2 and 3, at Level 2 they can access, act on, and use information and evaluate simple claims using data for tasks set in a variety of authentic contexts, interpreting the information presented in the task, and using multistep processes to solve it. At Level 3, this adult may be able to reflect and evaluate more complex situations, including authentic mathematical contexts, making judgements about how to use the given information, and being able to manage more complex situations where the information is less explicit and is embedded in contexts that not always are commonplace.

The Development From Level 3 to Level 4 and Above in Cognitive Processes

- At Level 3, adults can access, act on, reflect on, and evaluate authentic mathematical content. For Level 4, there is an additional focus on critical thinking when reflecting on and evaluating a broad range of mathematical information.
- At Level 3, adults make judgments about situations where mathematical information is less explicit, embedded in contexts that are not always commonplace, and use representations that are more formal and involve greater complexity. For Level 4, the situations are unfamiliar contexts, and the information may not be presented explicitly.
- At Level 3, mathematical processes require applying two or more steps and satisfying multiple conditions. For Level 4, adults can devise and implement strategies to solve multistep problems, which may involve reasoning about integrating concepts from different mathematical content areas or applying more complex and formal mathematical procedures.

Thus, when adults are transitioning between Levels 3 and 4, at Level 3 they can access, act on, reflect on, and evaluate authentic mathematical content embedded in contexts that are not always commonplace and use representations that are more formal and involve greater complexity. At Level 4, they should be able to think critically, reflect on, and evaluate a broad range of mathematical information presented in unfamiliar contexts and in non-explicit manners. Adults at Level 4 are expected to integrate concepts from different mathematical content areas or apply more complex and formal mathematical procedures.

PIAAC Mathematical Content Dimension

> What emerged from this study, exposited in Why Numbers Count, is that unlike mathematics, quantitative literacy is characterized by the use of simple quantitative tools to deal with complex issues. (Steen & Madison, 2011)

In traditional mathematics education, the difficulty of any given task is often viewed as being driven by the complexity of the mathematics content involved. Complexity was classified through the level of formality, sophistication and abstraction of the mathematical concepts and the difficulty of the calculations or mathematical processes that needed to be performed. Numeracy, however, deals with how individuals can manage daily, community, civic and workplace situations. Therefore, besides such mathematical complexity, other features are relevant, such as the content, the purpose, the thinking and reasoning processes involved in relating the situation to the relevant mathematics, and how the situation is represented or presented. The categories most commonly used in the PISA frameworks and many curricula worldwide are also used to categorize the relevant mathematical content in the PIAAC assessment framework.

In PIAAC, these mathematical categories are

- quantity and number,
- space and shape,
- change and relationships, and
- data and chance.

These categories don't drive the complexity of a situation per se. There is no evidence or reasoning to assume that one category is more difficult than another.

From analyzing the tasks and the data of the participant's performance, one can say that the most likely underlying factor that drives complexity is the number of "thinking steps" individuals must take in a situation to come to the desired result. These thinking steps can include completing calculations, solving a problem, finding an interpretation, or reasoning to cope with a situation. The mathematical content will play a role in those thinking steps and in solving a problem. As an adult progresses up to the higher levels of PIAAC numeracy proficiency, the sophistication and extent of abstraction of the mathematical concepts rise, but it is not the dominant factor in the complexity of the task.

Conversely, an additional factor to consider when evaluating mathematical content is the various forms in which it is presented. Mathematical content may be conveyed through textual or symbolic representation, but it is also frequently integrated within dynamic applications. In PIAAC Cycle 2, four classifications are delineated to represent numeracy within real-life contexts: text or symbols, images of physical objects, structured information, and dynamic applications. These representational aspects of difficulty are discussed in the following section.

Task complexity is not solely determined by the level of abstraction within the mathematical content. For instance, complexity might involve the use of units, tenths, hundredths, and various magnitudes; types of numbers; identifying numerical trends and regularities; understanding and employing multiple representations of numbers; number sense; the ability to comprehend and utilize measurement formulas, dimensions, and units; navigating different locations, directions, and perspectives; understanding geometrical properties such as symmetry and transformations; comparing, reading, and interpreting data presented in tables or graphs; employing algebraic expressions and related methods of solution; applying proportional reasoning, rates, and change; and grasping concepts of chance and probability.

Complexity is also shaped by the number of steps (involving different calculations or processes) required to arrive at an appropriate solution to the task or problem, the degree to which the adult is familiar with the situation or task, and the extent to which the information necessary to solve it is explicit or implicit. It is well established that the longer the sequence of operations required to solve a problem, the more challenging it becomes for an individual to comprehend the entire process, plan it, and execute it.

In the sections below, we summarize which increases in complexity in mathematics content were discernible, comparing the descriptions of the subsequent levels. We focused on the progression from Level 1 to Level 2, from Level 2 to Level 3, and from Level 3 to Level 4 and above.

Note: At the end of this section, we have included the descriptions of three PIAAC numeracy items that will help illustrate the progressions from level to level. When generalizing and summarizing the changes up the levels for a sample of over 70 items across Levels 1 through 5, it is difficult to define this range clearly. We hope the inclusion of some descriptions of actual items will help in the interpretation.

The Development From Level 1 to Level 2 in Mathematical Content

- Adults at Level 1 demonstrate number sense (comparing quantities) involving whole numbers, common and simple decimals, fractions and percentages (such as ½ or 10%). At Level 2, the numbers are slightly more complex (standard fractions and percentages such as ¾ 15%), and adults can reason with these numbers.
- Adults at Level 1 perform additions, subtractions, multiplications, and divisions with small whole numbers in straightforward situations, while at Level 2 these calculations can be with slightly more complex numbers. The use of a tool like a calculator will be more in demand at Level 2.
- Adults at Level 1 interpret simple spatial representations or a scale on a map, while adults at Level 2 also use tools to convert units or perform simple measurements.
- Adults at Level 1 identify a correct percentage in a given situation, while adults at Level 2 can perform simple calculations with percentages. Also at Level 2, adults use simple given formulas and evaluate them. Adults at Level 2 select multiple data from a diagram and categorize them or find the right category by given data.
- Adults at Level 2 make evaluations on simple straightforward statements.

Thus, when adults are transitioning between Levels 1 and 2, at Level 1, they can use numbers such as whole numbers, decimals and common fractions and percentages to count, compare quantities, perform basic operations (addition, subtraction, multiplication and division), interpret simple spatial representations and scale, and identify percentages in a given situation. At Level 2, they can also use higher numbers, understand their size (e.g. compare one thousand to one billion), use them to make some estimates (e.g., use the number of people standing up in a square meter to figure out the number of participants in a demonstration), to interpret and understand basic formulas (e.g. areas of regular polygons), and read, select and use data from simple tabular and graphical representations.

The Development From Level 2 to Level 3 in Mathematical Content

- Adults at Level 2 demonstrate number sense (comparing quantities) and perform simple calculations involving whole numbers, decimals, and standard fractions and percentages. At Level 3, the numbers are more complex, with multiple steps, various percentages, and choosing numbers given some conditions. Using formulas or calculation tools like calculators will be more common at Level 3.
- Adults at Level 3 make more extended or multiple combinations of thinking and calculating steps than at Level 2, and adults at Level 3 can make more complex calculations or mathematical processes than adults at Level 2, like calculations with time, averages, multiple variables, or with data from multiple sources or diagrams.
- Adults at Level 2 use tools to convert units or perform simple measurements, whereas adults at Level 3 reason, interpret, and then undertake the relevant calculations.
- Adults at Level 2 interpret two-dimensional representations (e.g., size of regular polygons), whereas adults at Level 3 can also interpret some three-dimensional representations (e.g., polyhedral and the most regular objects of revolution: sphere, cone, cylinder).
- Adults at Level 2 can substitute into simple given formulas and evaluate them, whereas adults at Level 3 use models and make or modify simple models or mathematical representations.
- Adults at Level 2 select multiple data from a diagram and categorize them or find the right category by given data, whereas adults at Level 3 interpret more complex data representations.
- Adults at Level 2 make evaluations on simple straightforward statements, whereas adults at Level 3 use multiple sources for interpreting and reasoning with data to check statements.

Thus, when adults are transitioning between Levels 2 and 3, at Level 2 they can perform simple calculations using a range of numbers (whole numbers, decimals, and standard fractions and percentages), use and convert units to do simple measurements, interpret and understand data represented in different formats (tabular or graphical), and use logical reasoning for simple situations. At Level 3, adults can combine different operations in a multistep line of reasonings and/or calculations, make more complex calculations (than just the basic arithmetic operations), manage three-dimensional objects, measure them (area and volume), and use multiple sources for interpreting and reasoning with data sets to check statements.

The Development From Level 3 to Level 4 and Above in Mathematical Content

- Adults at Level 4 and above use and apply a range of problem-solving strategies, much more than at Level 3.
- Adults at Level 4 and above critically reflect on and evaluate a broad range of mathematical information that is often presented in unfamiliar contexts. At Level 3 such information may be presented in an explicit manner, but at Level 4 and above it may be presented in a more implicit way across different sources.
- Adults at Level 3 reflect on and use mathematical reasoning when reviewing and evaluating the validity of conclusions drawn from data, including a limited set of related conditions or statements, and evaluate claims and stated relationships using a variety of data sources. Adults at Level 4 and above can even devise a strategy to compare large data sets and read and interpret multivariate data presented in a single graph.

Thus, when adults are transitioning between Level 3 and Level 4 and above, at Level 3 they can use problem-solving strategies (e.g., such as trial and error, look for patterns, analogy, make a representation, diagram or scheme, generalize, start from the end, etc.), reflect on explicit mathematical information, use mathematical reasoning to assess and evaluate the validity of conclusions draw from data. Adults at Level 4 and above can use a wider range of strategies to solve a problem, move easily from one to another, devise and perform different manners to read and interpret multivariate data presented in tables or graphs, and identify, understand and manage embedded mathematical information to achieve suitable answers to a given task or situation.

PIAAC Representation Dimension (and Related Literacy Aspects)

The role of text, literacy, representational, and the interpretative aspects of solving a numeracy task has been recognized as an issue for a long time but is still something not necessarily acknowledged and recognized as being a significant part of numeracy and mathematics teaching, learning, or instruction, nor has it been adequately researched (e.g. see Meiers & Trevitt, 2010; Newman 1977a, 1977b; Tout, 1991). This issue was explicitly addressed in the development of the PIAAC Numeracy Task Complexity Schema (Tout et al., 2020) where two of the factors addressed literacy or text-based aspects of solving a numeracy task.

In *Words and Worlds* (Verschaffel et al., 2009), the authors analyze the long history of inauthentic word problems in school maths textbooks and reveal the ambiguity of such tasks. On the one hand, the task pretends it models a situation in the real world, while on the other hand, the task demands a straightforward calculation with numbers given in the text and one right answer, which can only be obtained by ignoring all kinds of details in reality. The authors suggest that often the best way to solve word problems in school situations is to engage in a form of "suspension of sense-making."

In the PIAAC framework and its related numeracy assessments this problem is addressed by ensuring that the contexts used are in fact derived from real-world situations that exist outside of the mathematics classroom and are not manufactured, inauthentic situations.

Newman (1977a, 1977b) developed a structured method of error analysis related to the stages of solving a word problem. There were five stages, with the first three addressing the literacy, representational, and interpretative aspects of solving a task:

1. reading the problem,
2. comprehending the problem in your own words, and
3. transforming the problem from the real world to the mathematics world by identifying the mathematics that will be used.

These were then followed by two stages related to doing the mathematics and then taking the mathematical outcomes back into the real world:

1. process skills of actually carrying out the mathematics correctly, and
2. encoding the result from the mathematics world back to making sense of it in the real world.

Newman's (1977a, 1977b) stages are consistent with the PIAAC's cycle of cognitive processes. The first three stages (reading the problem; comprehending the problem in your own words; and transforming the problem from the real world to the mathematics world) parallel the *Access and assess situations mathematically* cognitive process. The fourth stage (process skills of actually carrying out the mathematics) parallels the *Act on and use mathematics* cognitive process, whilst the fifth stage (encoding the result from the mathematics world back to making sense of it in the real world) matches the third PIAAC cognitive process of *evaluate, critically reflect, make judgments*. This is also consistent with PISA's problem solving cycle of formulate, employ, interpret and evaluate (OECD, 2013).

Newman discovered from close study and interviews with school students that more than 50% of the barriers to numeracy performance were related to the first three of the five stages, before any formal mathematics process is employed. This result has been replicated in other countries and settings (White, 2009).

> Newman's research consistently pointed to the inappropriateness of many remedial mathematics programs in schools in which the revision of standard algorithms was overemphasized, while hardly any attention was given to difficulties associated with Comprehension and Transformation. (Ellerton & Clements, 1996, p. 186)

As illustrated, the literature on the complexity of numeracy shows that the ease (or difficulty) with which the embedded mathematics can be identified within a situation. This is particularly true when the situation is contextualized—such as in a real or authentic scenario—and not presented in an abstract, school-based classroom or textbook manner as a mundane assignment of simply using an algorithm.

Understanding the situation and discerning the type of calculation (or sequence of operations) necessary to reach an appropriate answer can sometimes be challenging. An individual might possess the knowledge to apply proportional reasoning but recognizing that proportional reasoning is required to solve a particular task is a different matter, particularly when the question is not explicitly stated in the task's description.

The implication of this for this paper is that it is important to recognize that the way the mathematics is represented, and the literacy and textual elements of how the mathematics contents are embedded in materials and stimuli are critical to address. This has been made explicit not only in the dimension of representation in the framework but also in the way these literacy aspects are incorporated into the first stage of the cognitive process: *access and assess situations mathematically.*

However, while there is an overlap between these two dimensions, this dimension of representation was separately analyzed and the results are detailed below and in Appendix C. The overlap between these two aspects of the analysis will be integrated when the outcomes are analyzed and summarized in the conclusions section.

The Development From Level 1 to Level 2 in Representational and Literacy Aspects

- At Level 1, adults can read and interpret mathematical information that uses simple, familiar and non-formal language and symbols and that can incorporate some more complex representations. The information is set in authentic and mainly familiar and commonplace contexts, and where the mathematical content is explicit and uses informal mathematical terminology with little text and minimal distracting information. For Level 2, information can be presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, multiple charts, or linear scales) that use some more formal terminology/language/symbols. The information can be less familiar or common, can be partially embedded although still quite explicit, can be located within a number of sources, and will have some distracting information. At Level 2, it may also be required to interpret representations and visualizations, and related text, where some translation is required.

- At Level 1, adults can respond to closed questions such as identifying or locating straightforward information and can respond utilizing some level of interactivity, whereas at Level 2, adults can respond to less closed questions that can require a level of interpretation and can respond by utilizing different levels of interactivity, such as locating and clicking on relevant information.

When adults are transitioning between Levels 1 and 2, at Level , adults can read and interpret mathematical information that is explicit and set in mainly familiar and commonplace contexts based on simple, familiar, and nonformal language and symbols. The tasks are usually simple, closed questions, and can require some level of interactivity, whereas at Level 2, the information can be less familiar or common, presented in slightly more complex forms, use some formal language/symbols, and be partially embedded although still quite explicit. At Level 2, it may also be required to interpret representations and visualizations, and adults can respond to less closed questions that require a level of interpretation and can respond by utilizing different levels of interactivity.

The Development From Level 2 to Level 3 in Representational and Literacy Aspects

- At Level 2, adults can read and interpret information presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, multiple charts, or linear scales) that use some more formal terminology/language/symbols where some interpretation is required. The information can be less familiar or common and can be partially embedded although still quite explicit and can be located within a number of sources and will have some distracting information. At Level 2, it may be required to interpret representations, visualizations, and related text, where some translation is required, whereas for Level 3, adults can read and interpret information, representations, and terminology that are more formal and involve greater mathematical complexity, including algebraic representations and conventions, and that are less explicit and based in contexts that are less familiar and can require a more formal mathematical interpretation or understanding. The information can be embedded within a number of sources within the text/activity, often including some distracting information. They may also need to bring to the problem-related information or knowledge from outside the problem.

- At Level 2, adults can respond to fairly closed questions that often require a level of interpretation and can respond by utilizing different levels of interactivity – such as locating and clicking on relevant information. For Level 3, adults can respond to more open/interpretive questions including satisfying different criteria and can respond by utilizing different levels of understanding and interactivity with a more technical, dynamic representation and visualization where interaction and interpretation are required (e.g., spreadsheet processes).

Thus, when adults are transitioning between Levels 2 and 3, they need to be able to progress to reading and interpreting information that is more formal and involves greater mathematical complexity, that are less explicit and less familiar, and where the information can be embedded within and across a number of sources with some distracting information. They may also need to bring to the problem-related information or knowledge from outside the problem. They also need to progress from only being able to answer fairly closed questions to being able to respond to more open/interpretive questions including satisfying different criteria and applying different levels of understanding and interactivity with more technical, dynamic representations.

The Development From Level 3 to Level 4 and Above in Representational and Literacy Aspects

- At Level 3, adults can access, act on, and use information, representations, and terminology that are more formal and involve greater mathematical complexity, including algebraic representations and conventions, and that are less explicit and based in contexts that are less familiar and can require a more formal mathematical interpretation or understanding. The information can be embedded within a number of sources within the text/activity, often including some distracting information. They may also need to bring to the problem some information or knowledge from outside the problem. Moving to Level 4 and above, adults can now move on to reflecting on and evaluating a broader range of mathematical information that is often presented in unfamiliar contexts, where the mathematics uses complex and formal mathematical terminology and representations including algebraic representations and conventions. The information, and the actions required, will not be as accessible or explicit, being more highly embedded alongside distracting information, where there is a need to interpret or translate the text and task.

- At Level 3, adults can respond to more open/interpretive questions including satisfying different criteria and can respond by utilizing different levels of understanding and interactivity with a more technical, dynamic representation a visualization where interaction and interpretation is required e.g., spreadsheet processes. At Level 4 and above, adults can now respond to more complex, open-ended tasks and use and apply more formal, technical representations or processes where some dynamic interaction and interpretation are required (e.g., understanding and use of spreadsheet processes), and also bring to the problem mathematical knowledge, reasoning and processes from outside the problem.

Thus, when adults are transitioning between Level 3 up to Level 4 or higher, they need to be able to progress to reading and interpreting information that is more formal and involves greater mathematical complexity, where the mathematical information can be highly embedded within and across a number of sources with distracting information and the actions required will not be explicit where there is a need to interpret or translate the text and task. They also need to progress to being able to respond to more complex, open-ended tasks and use and apply more formal, technical representations or processes where some dynamic interaction and interpretation are required.

Summary

It appears that the difficulty in relation to representation is not driven by the type of representation (text or symbols, images of physical objects, structured information, or dynamic applications). In numeracy at all levels from 1 through to 4, all types of representations occur. Difficulty seems to be driven more by factors relating to the complexity and familiarity of the text and context, and the complexity of the representations within each of the types. This includes key aspects such as

- the explicitness of mathematical information and
- the complexity and formality of how the mathematical information is provided: the language, terminology, representations, symbolism, and conventions used.

As well, this analysis identified that the complexity of the task, or what and how the adult needs to respond, impacts on the progression up the levels. This includes the type of question or task asked—is it straightforward and closed or more open and incorporates questions where interaction and interpretation is required of both the text and the task.

Intersections With Reading Literacy

As we have argued, numeracy involves solving a problem wherein some mathematics are embedded in an authentic real-world situation or context. In PIAAC, the numeracy assessment attempts to replicate this for the general population. As such, resolving and answering the question requires the adult to first decipher and interpret the information provided (in photos, pictures, words, graphs and charts, diagrams, maps and various combinations of these). Moreover, the process of finding a solution requires the application of a range of knowledge and skills including reading and interpreting the provided materials, using and applying different math skills, alongside reasoning. The PIAAC framework, its description, and definition of numeracy and numerate behavior attempt to elaborate on this as the basis for the assessment content. Thus, any assessment of numeracy contains elements of reading, meaning there are parallels and overlaps between the two skills.

> For adult numeracy learners, this complex set of linkages and interdependencies between numeracy and mathematics and literacy and language, points to a range of different sources of vulnerability that need to be understood and explicitly addressed in adult numeracy education. This also has consequences for the diagnosis of the stumbling blocks encountered by each individual adult numeracy learner, since they may have to do with

mathematics, or literacy, or language, or their different combinations. (Gal et al., 2020, p. 381)

This overlap and combination of skills is particularly important in the first stage of the PIAAC cognitive dimension (access and assess situations mathematically) where the initial stages of solving a numeracy problem require the reading and understanding of any text that is presented and reading and interpreting the task and question(s) being asked. This aspect is minimized in the Below Level 1 questions, especially in the numeracy component items. The reverse is also true—within reading there will be texts and materials that include numerical and mathematical or statistical terms and representations (words, graphs and charts, tables, diagrams for example), which will require elements of mathematical or statistical understanding.

Many of the different dimensions that are considered and elaborated in the accompanying Literacy paper that look at text and task features and complexities, and the interactions between text and task, will similarly apply to numeracy tasks and related performance. Real-world numeracy tasks, especially at Level 2 and above are based on texts and stimuli and as with reading, require interpretation and comprehension skills and the need to make inferences.

So, in developing strategies and interventions for supporting learners to improve their literacy and numeracy skills, it may make sense to integrate any literacy and numeracy instructional interventions and provide support to learners based on the tasks they are required to undertake in their lives. In the workplace, and life in general, the knowledge and skills of reading and mathematics are integrated; they are not skills or activities that can be separated out and undertaken independently of the other.

Description of Sample Numeracy Items at Levels 2 and 3

Three sample numeracy items are described below. Each is associated with a different text, as is quite common for the PIAAC numeracy tasks.

Level 1 Item (Bike Tour C801P001 / RP67 = 198 / a parameter = 1.34)

A very short visual advertisement for a bike tour is presented that consists of a list showing the number of kilometers that bikers will ride each day on a 3-day tour. The numbers are all two-digit whole numbers. Respondents are asked the number of kilometers in the complete tour. They must determine that “complete” means that they need to add the three provided distances in order to compute the total. This task is quite easy because the provided text is very simple and in a fairly commonplace context, the task is quite clear, and the set of three numbers in the advertisement is the only information that needs to be located and acted upon.

Level 2 Item (Urban Population N650P001 / RP67 = 260 / a parameter = .83)

This item is based on two pie charts showing the percentage of the world population living in rural areas, urban areas with less than 1 million people and urban areas with more than 1 million people in two different years. Respondents are asked what percentage of the world population was living in urban areas in one specified year. This item is rather challenging for a number of reasons. The text includes pie charts with labels that must be carefully read in order to notice that data for two categories of urban areas are included. The task does not specify that the total urban population is required, so respondents must use the information in the pie charts to recognize that two percentages must be located and added. And the task-by-text feature that adds to the difficulty is the presence of distracting information, in that the same data is shown for two different years.

Level 3 Item (Orchestra Ticket N664P001/ RP67 = 307 / a parameter = 1.33)

The final example is based on a table showing prices for concert tickets. In each of three seating categories, the prices for both a single concert ticket and a season ticket that includes 6 concerts are shown. The final category, student seating, shows only the price for a single concert ticket. Respondents are asked what the cost for a student season ticket would be using the same formula. The formula for calculating the discount for season tickets is not provided. This is a difficult item both because respondents must recognize what the task requires (algebraic thinking and reasoning) and because the required multistep calculations are rather complex.

Conclusion

The level of performance in numeracy situations is influenced by multiple interrelated factors, including various cognitive processes, pertinent mathematical content, and the way in which the situation is represented or presented. These factors are intertwined and overlapping, collectively shaping an individual's interaction with a given scenario. It is assumed that each adult has developed a distinctive mental mathematical toolkit, which they employ to navigate and manage numeracy-related situations.

In our analysis, we investigated the key aspects of the aforementioned factors, connecting them to the dimensions outlined in the PIAAC Numeracy Assessment Framework and the PIAAC Numeracy Task Complexity Schema.

In summary, the complexity of the whole process of managing the situation is largely determined by the challenges of accessing and interpreting the mathematics embedded within the context. This includes the necessity to take multiple steps, integrate information across different processes, utilize various sources, and meet multiple conditions or criteria. The complexity of the mathematical content as such seems to play a more modest role.

This implies that the elements of complexity related to comprehending the situation and determining the appropriate mathematical procedure to arrive at a satisfactory answer, decision, or action—such as identifying the relevant operation, or interpreting and reasoning with data—are indeed critical. These factors are fundamental in progressing from one level of numeracy performance to a more advanced one.

The PIAAC Cycle 2 data gave us a unique opportunity to look in a detailed way at all these factors and made it possible to differentiate between the performance typical for each level, as they are described and constructed in the PIAAC scale. This gives a more solid base to think about what are suitable interventions that would support and enable individuals to progress from one level to a higher level.

The following section discusses how the above findings alongside key aspects of adult learning and proposes a number of suitable interventions to help adults navigate the required learning trajectories from lower levels of numeracy performance to higher levels.

Interventions

This working paper has focused on utilizing the information we have available from the data collected through Cycle 2 of PIAAC on the overall proficiency of adults to answer the comprehensive set of questions and tasks included in the cognitive assessment.

While this analysis provides a good basis for analyzing which numeracy and mathematical knowledge and skills should be improved to support adults to transition from level to level, it needs to also be acknowledged that there are a range of other aspects which impact and drive the effectiveness of educational interventions aimed at making a transition to a higher level of numerate behavior.

To start this discussion, however, we want to comment on the issue of research, and associated knowledge, about the teaching and learning of adult numeracy, and not, as has been the focus up to now, of the assessment and descriptions of adults' proficiency in numeracy as measured by PIAAC Cycle 2. However, although some of the following has been discussed above, the purpose in this section is to consider what educational interventions can work to support the targeted adults to progress up a level on the PIAAC proficiency scale.

Dearth of Research on Adult Numeracy

Research about the teaching and learning of numeracy (or mathematics) for adults is very limited, and this has been the situation for a long period of time. For example, in a 2002 analysis, Tout and Schmitt found that while there was a high level of research into both mathematics education (in schools) and into adult basic education in the United States, less than 1% of any such reported research addressed numeracy or mathematics within adult basic education. In a very recent research survey, Gal (2024) found a similar dearth of research into adult numeracy teaching:

> The results show that only 39 relevant empirical studies were found among over 2,300 research papers reviewed, and that few of those focus on practice-

> related of adult education in mathematics and numeracy. The results provide quantitative evidence suggesting that the field of adult numeracy education is under-researched, and help to identify gaps in empirical research involving adult numeracy, including on emerging topics such as on modelling and critical interpretation. (p. 293)

In an OECD analysis, Windisch (2016) concluded that few intervention studies across OECD countries have been conducted and little is known about strategies that are most effective in improving adult literacy or numeracy skills. This study argued that tackling serious literacy and numeracy weaknesses is challenging because low-skilled adults are diverse and require different, well-targeted interventions.

In a much more recent survey paper (Gal et al., 2020), the authors recommended a number of research-worthy topics, given the scarcity of adult numeracy research. These included, amongst others:

- What is the impact of dispositional and affective factors (broadly viewed, including attitudes, purposes and motivations for learning, beliefs, identity, etc.), which may have both negative and positive aspects, on participation, retention, persistence, and engagement by learners of numeracy?
- What content knowledge and pedagogical content knowledge are required of teachers in the area of adult numeracy, and what professional development schemes can promote such knowledge?
- What curricula or teaching practices can enhance literacy-numeracy linkages, i.e., enable learners to effectively engage with the literacy aspects of numeracy? (Gal et al., 2020, p. 391)

What Is Known From Reviews of Adult Numeracy Teaching and Practice

However, there have been a number of theoretically based reviews and the development of professional development guides, curriculum frameworks, and more, about what practice should look like based on different perspectives and especially on practical experiences in the teaching and learning of numeracy and mathematics by adult learners. Much of this work was based around knowledge and understanding of adult learning theories and principles (for example, see Foley, 1995; Kidd, 1978; Knowles, 1980; Mackeracher, 1996; Rogers, 1996) in conjunction with understandings about the teaching and learning of mathematics to children, including related to problem-solving, mathematical modelling or contextualized teaching and learning. For example, Howey (1998) claimed that contextual learning emphasizes higher-level thinking, knowledge transfer, collecting, analyzing, and synthesizing information and data from multiple sources and viewpoints.

Much of this initial work happened in the 1990s, and we will summarize a few of these to start with, as further background to potential teaching practices on which to base any recommended interventions for improving the numeracy competence of existing adult learners.

In one example, Tout & Johnston (1995) developed an 84-hour national training program for teachers, titled *Adult Numeracy Teaching: Making Meaning in Mathematics* based on a set of theoretical underpinnings. The course was an intensive, hands-on program, where the outcomes stated:

> Teachers will gain a critical appreciation of the place of mathematics in society and be able to initiate appropriate learning activities by identifying the numeracy needs of students and responding with a variety of approaches, a breadth of knowledge, and appropriate mathematical resources. (Tout & Johnston, 1995, p. 4)

Within that broad purpose, this adult numeracy teaching program addressed a range of specific teaching practices based on different theoretical perspectives, and included, amongst others:

- Participants will identify and analyze concepts of mathematics and numeracy and how people learn mathematics; and identify their own assumptions about what mathematics is, how they and others learn it, how they feel about it and why, and how this affects the teaching and learning of mathematics in a range of adult basic education contexts.

- Participants will examine the role that mathematics plays in conveying information, and the interplay between mathematics, language, context and the political, social and cultural contexts within which the mathematics arises.
- Participants will develop an understanding that numeracy is not a subset of mathematics; that it is a critical awareness that
  - enables individuals to bridge the gap between mathematics and the diverse realities of their life and
  - is reflected by learners in their social practice. (Tout & Johnston, 1995, p. 4)

Around a similar time, Ginsburg & Gal (1996*)* undertook a project to investigate and develop a set of instructional strategies that addressed issues of assessment, development of mathematical skills, and development of problem-solving skills for dealing with real life contexts in which there was a math component. They produced 13 strategies based on a broad conception of what numeracy education encompassed and that supported ways to bring about meaningful and useful numeracy learning. Their list included the following:

1. Address and evaluate attitudes and beliefs regarding both learning math and using math.
2. Determine what students already know about a topic before instruction.
3. Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations and hands-on activities.
4. Encourage the development and practice of estimation skills.
5. Emphasize the use of “mental math” as a legitimate alternative computational strategy and encourage development of mental math skill by making connections between different mathematical procedures and concepts.
6. View computation as a tool for problem solving, not an end in itself.
7. Encourage use of multiple solution strategies.
8. Develop students’ calculator skills and foster familiarity with computer technology.

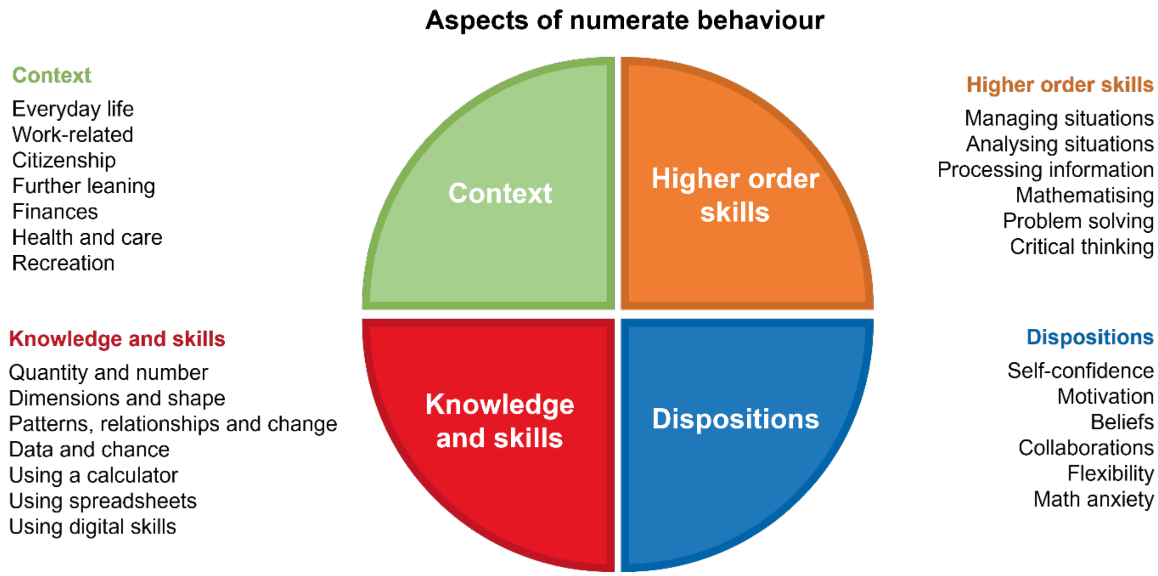
9. Provide opportunities for group work.
10. Link numeracy and literacy instruction by providing opportunities for students to communicate about mathematical issues.
11. Situate problem-solving tasks within meaningful, realistic contexts in order to facilitate transfer of learning.
12. Develop students' skills in interpreting numerical or graphical information appearing within documents and text.
13. Assess a broad range of skills, reasoning processes, and dispositions, using a range of methods.

In a later example, Maguire and O'Donoghue (2003) reviewed and organized conceptions of numeracy on a continuum of increasing levels of complexity or sophistication from formative, to mathematical through to integrative. They described formative conceptions as viewing numeracy as related to basic arithmetic skills, whereas mathematical conceptions consider numeracy in a contextualized way, as a broader set of mathematical knowledge and skills that go beyond basic computations and are relevant to everyday life. Third, they saw Integrative conceptions of numeracy as being a multifaceted, sophisticated construct incorporating not only mathematics but also communicative, cultural, social, emotional, and personal elements which interact and pertain to how different people function in their social contexts.

And, as mentioned earlier, Ginsburg et al. (2006) concluded that there were these key, integrated components of numeracy development: Context, Content, and Cognitive and Affective processes.

In addition, a recent project developed in Europe, funded by the European Commission, by Hoogland and his collaborators (Hoogland, 2023; Díez-Palomar et al., 2023), introduces a model of numeracy that reflects the multifaceted nature of this concept. This model is known as the Common European Numeracy Framework (CENF).

Figure 3. The Common European Numeracy Framework



From this perspective, numeracy is understood as the outcome of the interplay between four distinct dimensions. First, it encompasses mathematical knowledge and skills, which extend beyond basic arithmetic to include areas such as quantity and number, dimensions and shape, patterns, relationships and change, data and chance, as well as the application of technologies like calculators, spreadsheets, and other digital tools. Additionally, adults engage with mathematics through various cognitive processes, including managing or analyzing situations, processing information, mathematizing, problem-solving, and critical thinking. Moreover, mathematics is invariably applied within specific contexts. It is essential to consider that numeracy can manifest in everyday life situations, work-related contexts, or other domains such as citizenship, continued learning, finance, health and care, or recreation. Lastly, whether adults can access, act upon, and utilize information is influenced by a range of dispositions towards mathematics, including self-confidence, motivation, beliefs, collaboration, flexibility, and the presence (or absence) of mathematical anxiety. Collectively, these factors contribute to a complex reality that accounts for adult numeracy.

Many of the arguments in these, and other developments and conceptions of how to teach adult numeracy from the same or even later periods (e.g. Coben et al., 2007; Griffiths & Stone, 2013; Tout, 2017), espouse very similar approaches.

It should also be noted that research about adult numeracy practices, especially within workplaces, has found that most of the time adult learners do mathematics without being aware that they are doing mathematics because, for them, mathematics is what we call “formal or school mathematics,” not the nonformal or context-based mathematics embedded in authentic situations. Adults and workers see mathematics as invisible (e.g. see FitzSimons & Coben, 2009; Kent et al., 2007; Marr & Hagston, 2007; Wedege, 2010; Williams and Wake, 2006). Therefore, it is also important to address adults’ perceptions of mathematics and of its value and usefulness. Developing an awareness of the value of numeracy practices and empowering adults to see mathematics in a more constructive and positive way is part of an effective numeracy program.

Much of this portfolio of work from across the globe was moving toward an understanding that numeracy incorporated an Integrative conception of numeracy. And this is consistent with the description that the PIAAC Framework describes in its elaboration of numeracy, which is detailed in Appendix A, in particular in the section titled Elaboration of Numerate Behavior and Practices. Here it states:

> Specifically, the enabling processes involve integration of mathematical knowledge and conceptual understanding with broader reasoning, problem-solving skills, and literacy skills. Further, numerate behaviour and numerate practices and autonomous engagement with numeracy tasks depend on the dispositions (beliefs, attitudes, habits of minds, etc.), and prior experiences and practices that an adult brings to each situation. (OECD, 2021, p. 96)

Summary

What this means for this project and the recommendations in terms of any interventions, is that this Integrative perspective on what improving an adult’s numeracy knowledge skills and behaviors entails means that a complex, multifaceted, multidisciplinary approach needs to occur. Not only are there the set of cognitive related factors that we found in our research based on the PIAAC items and related construct, but also the need to acknowledge that a range of other noncognitive other factors play a significant and critical role.

We believe there is also a clear shift visible from a focus on rote learning and being able to perform the right calculation or follow a set process to using higher order skills, as described in the CENF framework mentioned previously. The need to teach higher order skills (modelling, problem solving, reasoning) is a theme that is occurring increasingly in research on mathematics education in general (for example, see Blum et al., 2007; Burkhardt, 2006; Törner et al., 2007). Similarly, in research about the adult numeracy and mathematics skills required in the 21st century, both from a work perspective, but also when taken from a societal view, it is recognized that the numeracy skills needed these days are higher order skills (for example, see Gal et al. 2020; OECD 2021; Tout et al., 2017).

Toward Recommended Teaching Practices for Any Intervention

Based on our analysis and the research in this working paper, we argue therefore that because numeracy is complex and is comprised both of cognitive elements (i.e., various knowledge bases and skills) as well as noncognitive or semicognitive elements (i.e., attitudes, beliefs, habits of mind, and other dispositions), any intervention strategies need to be based on the following perspectives:

- The situational aspects of where the training to be undertaken and its context/world knowledge and familiarity to participants. This should be the basis of the use of authentic materials and situations used in the training and of the numeracy-related practices implemented.
- The dispositions, beliefs and attitudes of the participants are critical and need to be incorporated and addressed explicitly
- That literacy, numeracy and mathematical skills are intertwined and should be integrated in any training and addressed jointly
- The different cognitive processes involved in solving a numeracy problem embedded in a real-world situation and as addressed and analyzed in this paper

- The complexity and formality of the mathematical information or data, including the type of operation or skills required
- The expected number of mathematical operations or processes
- The situational, and related textual and representational aspects of the embedded mathematics, including the explicitness or embeddedness of the mathematics

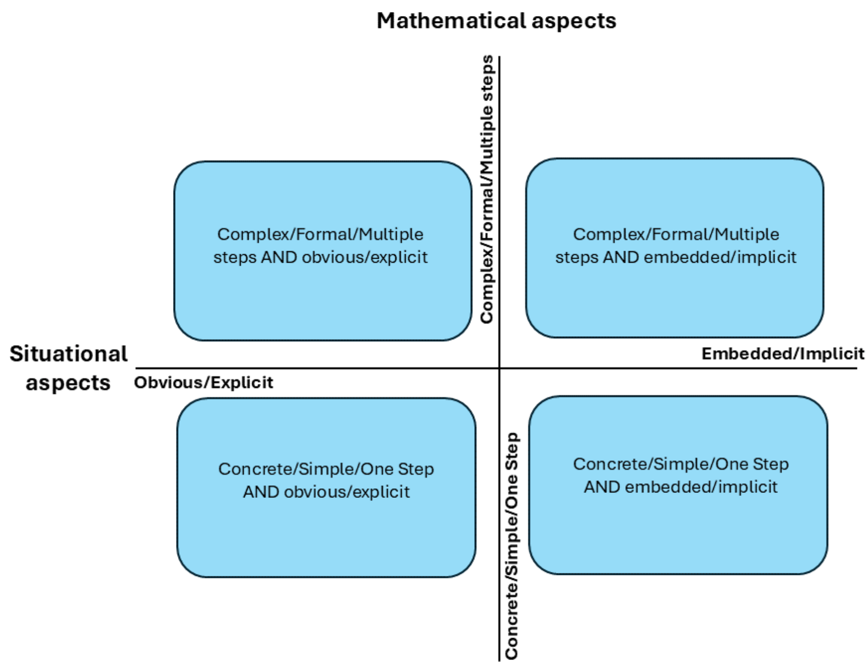
It is important to acknowledge the key factors contributing to complexity, both in terms of content and cognitive processes, including situational elements (such as the textual and representational aspects, the transparency of the problem, or the amount of distracting information) and the key mathematical elements. The representation below in Figure 4 can be utilized to visualize and assess a given task's complexity level.

The mathematical elements include the following:

- The complexity and formality of the mathematical information or data, including the type of operation or skills required, which can range from concrete and simple through to complex and formal mathematical content and processes
- The expected number of operations or processes which can vary from one simple step through to multiple steps

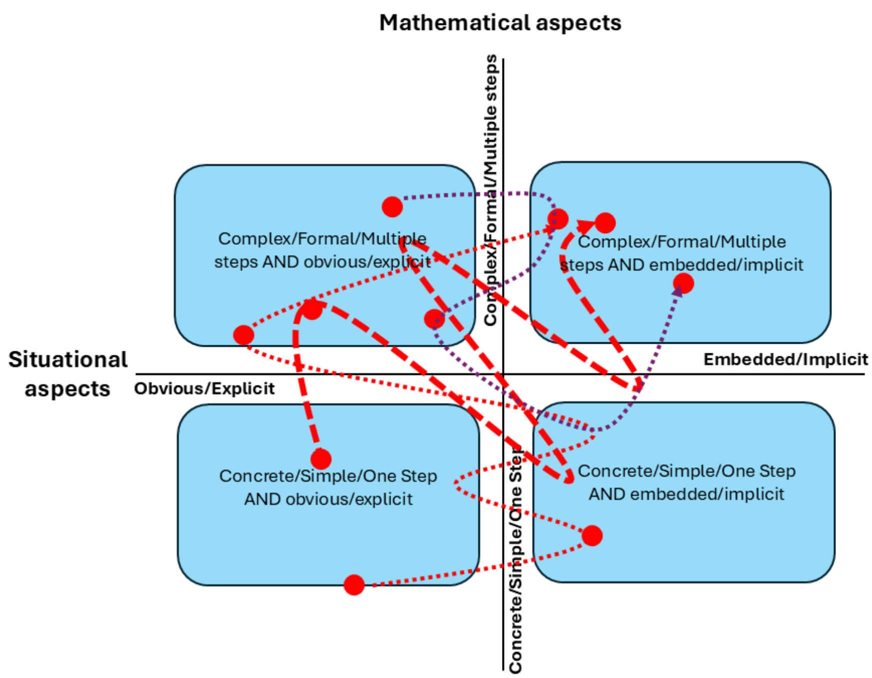
The situational, textual, and representational aspects relate to whether the embedded mathematics is obvious and explicit or whether it is highly embedded, and the mathematics is implicit.

Figure 4. Complexity Factors of Numeracy Rasks/Situations



Adults capable of solving Level 1 numeracy tasks in the PIAAC assessment would be positioned in the lower left quadrant of this diagram. In contrast, those able to solve tasks at level 4 or higher in the PIAAC assessment would occupy the upper right quadrant. Between these two extremes lies a spectrum of situations in which various adults would be positioned. Depending on their initial starting point or level, adults are likely to follow diverse trajectories in their learning process.

Figure 5. Different Potential Learning Trajectories



In adult education, learning trajectories are invariably diverse. Adults come with a wide range of school experiences as well as varied life experiences, particularly in numeracy situations, which are seldom uniform. Figure 5 illustrates different potential learning trajectories that adults may follow.

Underpinning Mathematical Content Foundations

When designing educational interventions, it is also important to acknowledge that in the PIAAC assessment adults are supposed to have some fundamental underpinning mathematical knowledge. There is a set of foundational mathematical cognitions that underpin the ability to increase the quality of an individual's numerate behavior. These include the following:

- Number sense, a basic recognition and feeling for quantity, basic dimensions and measurement and comparisons (how many, how big, everyday time events)
- Knowledge of some basic conventions: reading numbers, using symbols and words to represent numbers, use of the appropriate decimal separator (the decimal point or decimal comma)
- Basic ideas about the concepts of addition (+ subtraction) and multiplication (+ division) and their interrelationships
- Knowledge of basic shape properties, patterns and spatial relationships
- Knowledge of basic data representations

Our Recommendations

Our recommendations for educational interventions, which aim at a transition to a higher level of numerate behavior, are based upon our analysis of the data from the second cycle of PIAAC, but they are also consistent with contemporary literature on how to teach numeracy to adults in the 21st century as laid out in *Teaching Adult Numeracy – Principles and Practice* (Griffiths & Stone, 2013) and *Effective Teaching and Learning of Numeracy* (Coben at al., 2007).

In relation to appropriate strategies for building the necessary mathematical and numeracy skills and knowledge of any targeted individuals, the instructional practices should be based on the following:

- Address the adults' previous school experience and discuss possible feelings of math anxiety and how they are a result of school experiences

- Awareness of what counts as numeracy and numerate practices
- Make use of the previous experiences of the adults
- Value what adults already know about mathematics and make it visible; determine what they already know about a topic before instruction
- Use of authentic sources and materials taken from the situation(s) the adults are working and living with, and have adults gather examples from their own lives
- Ensure the mathematical knowledge and skills developed should arise out of the situation and the context, so any mathematical underpinnings are taught and developed only when needed ("Just in time" not "Just in case" as has often been the case in mathematics teaching and learning.)
- Use, as much as possible, a dialogical way of working, including developing any required mathematical understanding by providing opportunities to explore mathematical ideas with concrete or visual representations, hands-on activities and collaborative work with others.
- Link numeracy and literacy instruction, by making the role of language understanding and the reading and interpretation of texts explicit, including providing opportunities for students to communicate about mathematical issues.
- Include the development and practice of estimation skills.
- Emphasize the use of flexible and diverse ways of doing mathematical processes and calculations (e.g., in-the-head ways) as a legitimate alternative computational strategy by making connections between different mathematical procedures and concepts
- Stay away from a strict calculational or process driven approach
- Promote the use and application of a wide set of tools, both digital and analogue, emphasizing the efficient use of calculators and apps, spreadsheets, search engines and AI

- Focus on higher order skills like interpreting, mathematizing, reasoning, and reflecting critically

How this translates to specific interventions in the practice of numeracy education also depends on the beginning proficiency levels of the adults involved. In the section below, in a very general way, we show which points are the most relevant to focus on when aiming at a transition from one level to another.

However, this cannot be a standard, pre-defined program for any group of adults. Adults always have a very "spiky" profile, doing well in some parts of numeracy and mathematical domains but struggling with other parts. Initial assessments of each learner's skills and knowledge will need to be undertaken, alongside audits of the relevant situations, texts and materials and the related discourses and terminology in order to identify the underpinning numeracy, literacy and mathematical demands that will need to be incorporated into any intervention program.

Specific Focus Points for the Transition From (Below) Level 1 to Level 2

Some of the key aspects related to this transition include the following:

- Adults need to be able to progress to solving authentic numeracy problems where the situation, associated text, and mathematical representation
    - has some formal terminology/ language and representation;
    - is partially embedded and hidden, although still quite explicit;
    - is located within a number of sources with some distracting information; and
    - is less familiar or common.
- Adults need to be able to solve problems where the task or question requires undertaking one or two related steps and processes including interpretation and simple extrapolation.
- When adults are transitioning up to Level 2, they move to being able to use larger numbers, understand their size (e.g. compare one thousand to one billion), use it to

make some estimates, to interpret and understand basic formulae, and read, select and use data from simple tabular and graphical representations.

Specific Focus Points for the Transition From Level 2 to Level 3

Some of the key aspects related to this transition include the following:

- Adults transitioning to Level 3
    - move to being able to reflect and work with more complex situations, including more formal mathematical contexts, making judgements about how to use the given information,
    - move from interpreting to interpreting and reasoning, and
    - move from straightforward contexts to more complex contexts that are not always commonplace.
- Adults need to be able to progress to solving authentic numeracy problems where the situation, associated text and mathematical representation
    - uses more formal and complex terminology/ language and representation.
    - is embedded within a number of sources and are less explicit, and
    - is located within a number of sources, often including distracting information.
- Adults need to be able to solve problems where the tasks or questions asked are more complex,
    - where mathematical processes require the application of two or more different steps and
    - where multiple conditions or multiple sources may need to be accessed.
- When adults are transitioning to Level 3, they can
    - combine different operations in a multistep line of reasonings and/or calculations,
    - make more complex calculations (than the basic arithmetic operations),

- manage three-dimensional objects,
- measure them (area and volume), and
- use multiple sources for interpreting and reasoning with data sets to check statements.

Specific Focus Points for the Transition From Level 3 to Level 4 and Above

Some of the key aspects related to this transition include the following:

- Adults transitioning to Level 4, should be able
    - to think critically, reflect on, and evaluate a broad range of mathematical information presented in unfamiliar contexts and in nonexplicit manners,
    - to work with and across multiple sources, and
    - to undertake a broader range of thinking and calculating steps (including in planning and executing the steps and processes).
- They are expected to integrate concepts from different mathematical content areas or apply more complex and formal mathematical procedures.
- Adults need to be able to progress to solving authentic numeracy problems where the situation, associated text and mathematical representation
    - uses complex and formal mathematical terminology and representations,
    - is highly embedded where the mathematics can be implicit,
    - includes considerable distracting information
    - can have unfamiliar contexts,
- Adults need to be able to solve problems where the tasks or questions are more open, requiring interaction and interpretation across both the text and the task, with some analysis that involves applying of a number of processes, including critical reflection and evaluating.

- When adults advance to Level 4 and above, they begin to employ a broader range of strategies for problem-solving, seamlessly switching between methods. They also become adept at devising and executing various approaches to read and interpret multivariate data presented in tables or graphs, as well as at identifying, understanding, and managing embedded mathematical information to achieve appropriate solutions for a given task or situation.

References

Australian Association of Mathematics Teachers & Australian Industry Group. (2014). *Tackling the school–industry mathematics divide*. Commonwealth of Australia. https://www.aamt.edu.au/content/download/31676/446921/file/quantitative_skills_summary_v2.pdf

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., Miller-Ricci, M., & Rumble, M. (2012). Defining twenty-first century skills. In P. Griffin, B. McGaw, & E. Care (Eds.), *Assessment and teaching of 21st century skills* (pp. 17–66). Springer.

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (2007). *Modelling and applications in mathematics education - The 14th ICMI study.* Springer.

Brown, J., Taylor, M., McKenzie, P., & Perkins, K. (2015). *Investing in workforce literacy pays: Building employer commitment to workplace language, literacy and numeracy programs*. Australian Industry Group & ACER. https://research.acer.edu.au/transitions_misc/27

Burkhardt, H. (2006). Modelling in mathematics classrooms: Reflections on past developments and the future. *ZDM Mathematics Education*, *38*(2), 178–195. https://doi.org/10.1007/BF02655888

Clayton, N. (2019). *Evidence review: What works to improve adult basic skills? - July 2019*, Learning and Work Institute. United Kingdom.

Coben, D., Brown, M., Rhodes, V., Swain, J., Ananiadou, K., Brown, P., Ashton, J., Holder, D., Lowe, S., Magee, C., Nieduszynska, S., & Storey, V. (2007). *Effective teaching and learning numeracy.* http://www.nrdc.org.uk/?p=177

Díez-Palomar, J., Ramis-Salas, M., Močnik, I., Simonič, M., & Hoogland, K. (2023). Challenges for numeracy awareness in the 21st century: Making visible the invisible. In *Frontiers in Education*, *8*, Article 1295781. https://doi.org/10.3389/feduc.2023.1295781

Ellerton, N. F., & Clements, M. A. (1996). Newman Error Analysis: A comparative study involving year 7 students in Malaysia and Australia. In P. C. Clarkson (ed.), *Technology in mathematics education: Proceedings of the 19th annual conference of the Mathematics Education Research Group of Australia (MERGA).* UNESCO.

FitzSimons, G., & Coben. D. (2009). Adult numeracy for work and life: Curriculum and teaching implications of recent research. In *International handbook of education for the changing world of work*. Springer, Dordrecht https://doi.org/10.1007/978-1-4020-5281-1_179

Foley, G. (Ed.). (1995). *Understanding adult education and training*. Allen & Unwin.

Foundation for Young Australians. (2017). *The new basics: Big data reveals the skills young people need for the new work order*. Foundation for Young Australians. https://www.fya.org.au/wp-content/uploads/2016/04/The-New-Basics_Update_Web.pdf

Gal, I. (2024). Adult education in mathematics and numeracy: a scoping review of recent research. *ZDM Mathematics Education*, *56*(2), 293–305. https://doi.org/10.1007/s11858-024-01549-z

Gal, I., Grotlüschen, A., & Tout, D. (2020). Numeracy, adult education, and vulnerable adults: A critical view of a neglected field. *ZDM Mathematics Education*, *52*, 377–394. https://doi.org/10.1007/s11858-020-01155-9

Ginsburg, L., & Gal, I. (1996)*. Instructional strategies for teaching adult numeracy skills*. National Center on Adult Literacy, University of Pennsylvania. https://www.researchgate.net/publication/2821862_Instructional_Strategies_for_Teaching_Adult_Numeracy_Skills

Ginsburg, L., Manly, M., & Schmitt, M. (2006). *The components of numeracy* (NCSALL Occasional Paper). National Center for the Study of Adult Learning and Literacy, Harvard Graduate School of Education.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education*, *15*, 105–123. https://link.springer.com/article/10.1007%2Fs10763-017-9814-6

Griffin, P., McGaw, B., & Care, E. (Eds.). (2012). *Assessment and teaching of 21st century skills*. Springer.

Griffiths, G., & Stone, R. (2013). Teaching adult numeracy: Principles & practice. Open University Press.

Hanushek, E. A., Schwerdt, G., Wiederhold, S., & Woessmann, L. (2015). Returns to skills around the world: Evidence from PIAAC. *European Economic Review*, *73*, 103–130. https://doi.org/10.1016/j.euroecorev.2014.10.006

Hoogland, K. (2023). The changing nature of basic skills in numeracy. *Frontiers in Education*, *8.* https://doi.org/10.3389/feduc.2023.1293754

Hoogland, K., & Díez-Palomar, J. (2022). The mathematisation of society: Rethinking basic skills for adults. In *Twelfth Congress of the European Society for Research in Mathematics Education* (pp. 1250–1257). https://hal.archives-ouvertes.fr/hal-03745525v1

Hoyles, C., Noss, R., Kent, P., & Bakker, A. (2010). *Improving mathematics at work: The need for techno-mathematical literacies*. Routledge.

Kent, P., Noss, R., Guile, D., Hoyles, C., & Baker, A. (2007). Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind, Culture, and Activity, 14*(1–2), 64–82. https://doi.org/10.1080/10749030701307747

Kidd, J. R. (1978). *How adults learn*. Prentice-Hall.

Knowles, M. S. (1980). *The modern practice of adult education: From andragogy to pedagogy*. Cambridge Adult Education.

Lane, M., & Conlon, G. (2016). *The impact of literacy, numeracy and computer skills on earnings and employment outcomes*. OECD Publishing. https://www.oecd.org/content/dam/oecd/en/publications/reports/2016/03/the-impact-of-literacy-numeracy-and-computer-skills-on-earnings-and-employment-outcomes_g17a278d/5jm2cv4t4gzs-en.pdf

Mackeracher, D. (1996). *Making sense of adult learning*. Culture Concepts.

Maguire, T., & O’Donoghue, J. (2003). Numeracy concept sophistication - an organizing framework, a useful thinking tool. In J. Maass & W. Schlöglmann (Eds.), *Learning mathematics to live and work in our world*. *Proceedings of the 10th international conference on adults learning mathematics.* ALM.

Marr, B., & Hagston, J. (2007). *Thinking beyond numbers: Learning numeracy for the Future workplace.* NCVER. https://www.ncver.edu.au/__data/assets/file/0017/5426/nl05002.pdf

Meiers, M., & Trevitt, J. (2010). Language in the mathematics classroom. *The Digest*. NSWIT, Australian Council for Educational Research. http://www.nswteachers.nsw.edu.au

National Council of Teachers of Mathematics. (2017). *Catalyzing change in high school mathematics.* https://www.nctm.org/uploadedFiles/Standards_and_Positions/CatalyzingChangePublicReview.pdf

Newman, M. A. (1977a). An analysis of sixth-grade pupils' errors on written mathematical tasks. In M. A. Clements & J. Foyster (Eds.), *Research in mathematics education in Australia* (Vol. 2, pp. 269–287). Swinburne College Press.

Newman, M. A. (1977b). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin*, *39*, 31–43.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis & S. Papastavridis (Eds.), *Mediterranean Conference on Mathematical Education* (pp. 115–124). Hellenic Mathematical Society and Cyprus Mathematical Society.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*, 9–28. https://doi.org/10.1007/s10649-019-09903-9

OECD. (2013). *OECD skills outlook 2013: First results from the survey of adult skills*. OECD Publishing. http://dx.doi.org/10.1787/9789264204256-en

OECD. (2016). *Skills matter: Further results from the survey of adult skills* (OECD Skills Studies series). OECD Publishing. https://doi.org/10.1787/9789264258051-en

OECD. (2019). *Skills matter: Additional results from the Survey of Adult Skills* (OECD Skills Studies series). OECD Publishing. https://doi.org/10.1787/1f029d8f-en

OECD. (2021). *The assessment frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (OECD Skills Studies series). OECD Publishing. https://doi.org/10.1787/4bc2342d-en.

Partnership for 21st Century Skills. (2016). *Framework for 21st century learning*. Partnership for 21st Century Skills. http://static.battelleforkids.org/documents/p21/P21_Framework_Brief.pdf.

Pellegrino, J. W., & Hilton, M. L. (2012). *Education for life and work: Developing transferable knowledge and skills in the 21st century.* National Academy of Sciences.

Rogers, A. (1996). *Teaching adults.* Open University Press.

Steen, L. A., & Madison, B. (2011). Reflections on the tenth anniversary of mathematics and democracy. *Numeracy*, *4*(1), 1–6. https://doi.org/10.5038/1936-4660.4.1.1

Törner, G., Schoenfeld, A. H., & Reiss, K. M. (2007). Problem solving around the world: Summing up the state of the art. *ZDM Mathematics Education*, *39*(5–6), 353. https://doi.org/10.1007/s11858-007-0053-0

Tout, D., Coben, D., Geiger, V., Ginsburg, L., Hoogland, K., Maguire, T., Thomson, S., & Turner, R. (2017). *Review of the PIAAC Numeracy Assessment Framework: Final report*. Australian Council for Educational Research. https://research.acer.edu.au/transitions_misc/29

Tout, D., Gal, I., van Groenestijn, M., Manly, M., & Schmitt, M. J. (2020). *PIAAC Numeracy Task Complexity Schema: Factors that impact on item difficulty*. Australian Council for Educational Research.

Tout, D., & Johnston, B. (1995). *Adult numeracy teaching: Making meaning in mathematics.* National Staff Development Committee for Vocational Education and Training. https://files.eric.ed.gov/fulltext/ED392947.pdf

Tout, D., & Schmitt, M. (2002). The inclusion of numeracy in adult basic education. In J. Comings, B. Garner, & C. Smith (Eds.), *The Annual Review of Adult Learning and Literacy*, Vol. 3, pp. 152–202). Jossey-Bass.

Victorian Curriculum and Assessment Authority. (2021). *VCE and VCAL administrative handbook 2021.* https://www.vcaa.vic.edu.au/Documents/handbook/2021/VCEVCALAdministrativeHandbook2021.pdf

Victorian Curriculum and Assessment Authority. (2021). *VCAL assessment.* https://www.vcaa.vic.edu.au/assessment/vcal-assessment/Pages/Index.aspx

Victorian Curriculum and Assessment Authority. (n.d.). *Performance in senior secondary.* Retrieved May 12, 2025, from https://www.vcaa.vic.edu.au/administration/research-and-statistics/performance-senior-secondary/Pages/Index.aspx

Verschaffel, L., Greer, B., van Dooren, W., & Mukhopadhyay, S. (2009). *Words and worlds - Modelling verbal descriptions of situations*. Sense.

Vignoles, A., & Cherry, G. (2020). What is the economic value of literacy and numeracy? *IZA world of labor*. IZA Institute of Labor Economics. https://doi.org/10.15185/izawol.229.v2

Wedege, T. (2010). People's mathematics in working life: Why is it invisible? *Adults learning mathematics*, *5*(1), 89–97.

White, A. L. (2009). Diagnostic and pedagogical issues with mathematical word problems. *Brunei International Journal of Science and Mathematics Education*, *1*(1), 100–112. http://shbieejournal.files.wordpress.com/2009/11/awhite_2009bijsmepdf.pdf

Williams, J., & Wake, G. (2006). Black boxes in workplace mathematics. *Educational Studies in Mathematics*, *64*(3), 317–343. https://doi.org/10.1007/s10649-006-9039-z

Windisch, H. (2016). How to motivate adults with low literacy and numeracy skills to engage and persist in learning: A literature review of policy interventions. *International Review of Education*, *62*(3), 279–297. https://doi.org/10.1007/s11159-016-9553-x

APPENDIX A. Description of Numeracy for PIAAC Cycle 2

Extracts from the PIAAC Cycle 2 Framework (OECD, 2021)

Definition of numeracy for PIAAC Cycle 2

The PIAAC Cycle 2 definition for numeracy is:

Numeracy is accessing, using and reasoning critically with mathematical content, information and ideas represented in multiple ways in order to engage in and manage the mathematical demands of a range of situations in adult life.

Elaboration of numerate behaviour and practices[3]

This definition leads on to an elaboration of numerate behavior, which is included in Table 2.2 below. This table first lists the direct assessment components of the four core dimensions of the definition. Additionally, the bottom part of Table 2.2 also lists several non-cognitive, enabling factors and processes, whose activation underlies numerate behavior and successful numeracy practices. Overall, the definition of numeracy and the description of numerate behavior, with the details in Table 2.2 and the further explanations of the core dimensions following in the next section, provide the structure and roadmap for the development of the numeracy assessment as part of PIAAC Cycle 2.

Table A1. Numerate behaviour and practices – key facets and their components

Numeracy is an individual's capacity to **…**
1. ***access, use and reason critically***
- Access and assess situations mathematically (assess, identify, access and represent)
- Act on and use mathematics (order, count, estimate, compute, measure, graph and draw)
- Evaluate, critically reflect, make judgments (evaluate, reflect, justify and explain)
2. with ***mathematical content***
- quantity and number
- space and shape
- relationship and change
- data and chance
3. ***represented in multiple ways***
- Text or symbols
- Images of physical objects
- Structured information
- Dynamic applications.
4. in order to engage in and manage the mathematical demands of a ***range of situations*** in adult life**:**

- personal - work - societal/community.
An individual's numerate capacity is founded on the activation of several enabling factors and processes: - context/world knowledge and familiarity - literacy skills - disposition, beliefs and attitudes - numeracy-related practices and experience.

Enabling factors and processes

Adults' numeracy competence is revealed through their responses to the mathematical information or ideas that may be represented in a situation or that can be applied to the situation at hand. It is clear that numerate behavior will involve an attempt to engage with a task and not delegate it to others or deal with it by intentionally ignoring its mathematical content. Numerate behavior, however, depends not only on cognitive skills or knowledge bases, but also on several enabling factors and processes as listed in Table 2.2.

As outlined in the second section, including in the discussion about Numerate behavior and practices, the PIAAC conceptualization of numeracy operates on two levels. It relates to numeracy as a construct describing an individual's capability to solve numeracy problems, and also to numerate *behavior and practice* which is the way a person's numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature.

We argue therefore that numeracy as described in PIAAC is comprised both of cognitive elements (i.e., various knowledge bases and skills) as well as non-cognitive or semi-cognitive elements (i.e., attitudes, beliefs, habits of mind, and other dispositions) which together help to shape a person's numerate behavior and practices. Based on this, there are four non-cognitive or semi-cognitive enabling factors included in the elaboration of numerate behavior:

- context/world knowledge and familiarity
- literacy skills
- disposition, beliefs and attitudes
- numeracy-related practices and experience.

Specifically, the enabling processes involve integration of mathematical knowledge and conceptual understanding with broader reasoning, problem-solving skills, and literacy skills. Further, numerate behavior and numerate practices and autonomous engagement with numeracy tasks depend on the dispositions (beliefs, attitudes, habits of minds, etc.), and prior experiences and practices that an adult brings to each situation. These are briefly summarized below. Most of these enabling factors and processes have also been described by Kilpatrick, Swafford and Findell (2001) as part of his analysis of the construct of mathematical literacy, and further examined and deemed relevance for description of adult numeracy in the analysis by Ginsburg et al. (2006).

It should be noted that the direct assessment via the numeracy test component in PIAAC has as its primary emphasis the cognitive aspects of numerate behavior as framed in the first part of Table 2.2, namely the numeracy and mathematical knowledge and skills that underpin answering the test questions, which are mediated by written materials, without oral support, in the context of a formal assessment. The noncognitive aspects of numerate behavior are addressed indirectly through other components of the PIAAC assessment, namely through the skills use questions and the comprehensive background questionnaire.

The dimensions in PIAAC Cycle 2[4]

This sub-section elaborates on the dimensions incorporated into the definition of numeracy and the elaboration of numerate capacity. There are four core dimensions named and described in numeracy for PIAAC Cycle 2, namely:

- cognitive processes
- content
- representations
- context.

Each of these four core dimensions are elaborated below.

Cognitive processes

Description[5]

Solving problems in real-world contexts requires a range of capabilities and cognitive processes. When engaging with a real-world problem, one of the decisions to be made is whether the use of mathematics is relevant and then if it is best way to solve a problem. If the use of mathematics is deemed appropriate, the essential features of the problem will need to be identified in order to turn the real-world situation into a mathematical problem. From this point, relevant mathematical content, procedures, processes and tools needed to solve the problem must be identified and accessed by the problem solver. Once accessed, these procedures and processes will need to be employed correctly and decisions made about the appropriate degree of accuracy required to yield a mathematical solution. The solution needs to be reflected on and evaluated against the original problem situation in terms of its reasonableness and relevance to the real-world context and a decision made about whether to accept the solution or to revisit aspects that require refinement. In cases where decisions or judgements are being made on the basis of the solution, other factors might also be considered such as social or economic consequences.

- So, the first core dimension described in the PIAAC definition and elaboration of numeracy is about the cognitive skills and processes required to engage with and solve the task or problem at hand. These have been named as:
- access and assess situations mathematically
- act on and use mathematics
- evaluate, critically reflect, make judgments.

It is important to understand that these activities are not mutually exclusive of one another or that they take place in a rigidly linear manner. For example: the identification of a problem's essential features will have consequences for the identification of relevant mathematics to be engaged; an inability to access a particular area of mathematics may result in the selection of mathematical procedures and processes that are less effective; or the evaluation of the solution against the original problem situation may indicate those features, identified as essential, were not as relevant as first thought and so backtracking through the steps of the solution is necessary. Thus, while the cognitive processes outlined in this sub-section are described separately, the activity of addressing a real-world problem via mathematical means should be considered first and foremost as a holistic process.

It will be the combination of these three processes and their components that drive the difficulty and complexity of each numeracy problem being solved and each question asked in PIAAC numeracy units and items. After the description of each cognitive process below, there are a number of key questions outlined that describe the issues and factors that will influence the complexity of each process.

Note: for the purpose of guaranteeing a spread of types of items across PIAAC that focus on or emphasize the different aspects of these cognitive processes, each item has been prioritized against one of the three processes.

Access and assess situations mathematically (assess, identify, access and represent)

When adults encounter problems within real-world contexts they must first decide if mathematics is an appropriate means to engage with the situation. Once they deem the use of mathematics will provide advantage in addressing the problem, they need to identify the essential features to be accommodated when transforming the real-world situation into a mathematical problem. This transformation requires adults to look forward and identify and access the mathematics and mathematical representation embedded in the specifics of the situation, and make decisions about how the task can be represented and solved mathematically. The direction of the thinking and reasoning in this process is going from the real world to the mathematical world.

The actions that underpin assessing situations and accessing the mathematics in order to solve a real-world problem include:

- identifying the essential features of a real-world problem that can be represented mathematically
- identifying and describing/defining the mathematical operation(s), processes and tools needed to solve the problem
- simplifying a situation or problem in order to represent it mathematically, using appropriate representations, for example, variables, symbols, diagrams, and models
- representing a problem in a different way, including organizing it according to mathematical concepts and making appropriate assumptions

- anticipating the real-world restrictions on the possible outcomes of decisions made while defining and representing the problem.

Key questions that drive the complexity of this process:

- How is the mathematics represented and embedded within the real-world situation? Through words and language? Through numbers and symbols, diagrams, pictures, graphs and charts? How informal, formal or complex are the mathematical representations and the mathematical information?
- Is a mathematical approach suitable for the presented situation? Is the use of mathematics a sensible way to address the real-world problem? If so, what is the degree of transformation required of the real-world situation to move it into a mathematical problem? How implicit or explicit/obvious is it to decide on the mathematical problem-solving solution? Is the question presented in an unambiguous way so that necessary mathematical processes and procedures can be identified?
- What literacy skills are required to make this transformation – what are the reading demands, how much distracting information is there?
- Will a decision need to be made about how well the solution generated by solving the mathematical representation of the problem matches the contexts of the original real-world situation? How complex is that decision?

Act on and use mathematics (order, count, estimate, compute, measure, graph and draw)

Adults utilize mathematical processes, facts and procedures in order to derive results and solve real-world problems, and will need to select and use appropriate tools, including technology. For example, they may need to perform arithmetic computations; select, create, solve equations; make logical deductions from mathematical assumptions; perform symbolic manipulations; create and extract information from mathematical tables and graphs; represent and manipulate geometrical objects in 2D and 3D; and analyse data. Mathematical processes and procedures used to solve real-world problems include:

- applying mathematical facts, rules and structures

- performing arithmetic computations and applying routine algorithms
- undertaking measurements
- looking for a pattern
- using symbolic, formal, and technical language and mathematical conventions
- using mathematical tools, including technology
- manipulating numbers, graphical, statistical and chance-based data and information, algebraic expressions and equations, geometric representations
- collecting, organizing, structuring and representing information
- generating estimations and approximations
- making and extracting information from mathematical diagrams, graphs, info-graphics and constructions
- reviewing and reflecting upon initial or part solutions
- generalize from a more complex mathematical situation to a simpler mathematical problem/situation that can be more easily solved.

Key questions that drive the complexity of this process:

- How difficult and complex are the mathematical concepts, facts, processes and procedures that need to be used and applied?
- What level of mathematical reasoning, arguing, manipulating and computing is required for an effective response to the problem?
- How many steps and types of mathematical steps/processes are required to solve the problem? Is it one operation, action or process or does it require the integration of several steps covering more than one different operation, action or process?

Evaluate, critically reflect, make judgments (evaluate, reflect, justify and explain)

Responses to real-world tasks, including any mathematical solutions, judgements, decisions or conclusions, require reasoning and critical reflection and evaluation. Any solution of a real-world problem needs to be evaluated against the original problem situation in terms of its reasonableness and relevance to the original context and a decision made about whether to accept the solution or to revise and adjust the solution—often referred to as contextual judgement. In cases where decisions or judgements are being made on the basis of the solution, other factors might also be considered such as social or economic consequences. This will require that responses include explanations and justifications for decisions, judgements and conclusions that are reasonable and make sense within the context of the original situation. Critical reflection and evaluation within real-world contexts requires:

- evaluating the reasonableness of a solution or part solution to a problem. This includes consideration of the appropriateness of estimations and/or the degree of accuracy required
- understanding the real-world implications of solutions generated by mathematical methods, in order to critically reflect and make judgements about how the results should be adjusted or applied
- using mathematical arguments to construct, defend or challenge decisions and/or judgements
- considering social norms and influences, in addition to physical constraints, when considering the validity or effectiveness of a mathematical solution to a real-world problem
- reflecting on mathematical processes and arguments used and explaining and justifying results
- identifying and critiquing the limitations inherent in solving some real-world problems.

Key questions that drive the complexity of this process:

- How complex is it to evaluate, reflect, justify, and explain and connect the mathematical outcomes to the real-world context? Does the task require a choice from a number of provided solution options? Or does the task require an explanation to be derived or decided upon with no provided solutions?

- How complex is it to justify the validity of the mathematical outcomes and evidence with the essential elements of the original real-world problem? To what extent does the task require judgement about the quality of a mathematical argument used to defend or challenge a proposition within a real-world context?
- How complex is it to connect the mathematical evidence to the essential elements of the real-world problem? To what extent does the task require judgement about the appropriateness and reasonableness of a proposed result to the real-world context? To what extent does the mathematical result need to be adapted to fit in with the original real-world context? Does it require consideration of the appropriateness of estimations and/or the degree of accuracy required?

These three *Cognitive processes* are linked to the Numeracy Complexity Schema described further in the fourth section: *Operationalisation of the PIAAC Numeracy Assessment* and detailed in Tout et al. (2020). It is believed that the cognitive processes will drive much of the item difficulty and that together with the descriptions and scores described in the Complexity Schema, these will help to describe performance when it comes to elaborating the different levels in PIAAC.

Mathematical content

Description[6]

Four key areas of mathematical content, information and ideas are described and used in the numeracy assessment in PIAAC:

- Quantity and number
- Space and shape
- Change and relationships
- Data and chance.

For an individual item in PIAAC numeracy, these four content areas are not mutually exclusive and any item may involve one or more of these mathematical content areas. For example, a unit and item in *Data and chance* will necessarily also include data that will be expressed as a quantity or number, and similarly a measurement item in *Space and shape* will be expressed as a quantity or number. The classification of such items is based on what content area the key conceptual understanding and skill is directed at.

Quantity and number

The notion of quantity and number is a fundamental and essential mathematical aspect of engaging with, and functioning in, our world. The *Quantity and number* content area involves understanding ordering, counts, place value, magnitudes, indicators, relative size and numerical trends. This will encompass aspects of quantitative reasoning, such as number sense, multiple representations of numbers, computation, mental calculation, estimation and judging the reasonableness of results. This content area requires knowing and applying integers, rational and irrational numbers, positive and negative numbers and equivalence. It also requires understanding and applying number operations, including order of operations, in a wide variety of settings.

Illustrative examples:

- Identify and counting the number of items shown in a photo of a set of items or object.
- Calculating the cost of one can of soup, given the cost of 4.
- Calculating the cost when buying 0.283 kg of cheese at a given price per kg.

Another example could be deciding whether given decimal numbers are within a given range.

Space and Shape

The *Space and shape* content area encompasses a wide range of phenomena that are encountered everywhere in our visual and physical world. It includes an understanding and use of measurement (informal and standardized) systems, measurement formulas; dimensions and units; location and direction; geometric shapes and patterns; angle properties; symmetry; transformations and 2D and 3D representations and perspectives. This content area requires understanding and interpreting measurements and scales, position and orientation, plans, models, maps and diagrams, and navigation (including understanding travel distances, speeds and times, and using tools such as Global Positioning Systems).

Illustrative examples:

- The identification of a shape or matching an image of a real object to the correct plan/diagram.

- Reading the weight/mass of an object off an analogue scale.
- Interpreting an online map in relation to travel distances, speeds and times.
- Working out quantities required for a task such as wallpapering or tiling or painting given particular dimensions.

Change and Relationships

The *Change and relationship* content area includes the ways to describe, model and interpret mathematical relationships, quantitative patterns, and change, where they occur in the real world. Real-world variables can be based around linear and non-linear relationships. Such relationships can be represented by descriptions, picture or images, tables, graphs or formula. In the latter case it could require the understanding and use of algebraic expressions and related methods of solution. This content area requires understanding, using and applying proportional reasoning and rates of change, including the use and application of ratios. It also requires recognizing, describing, and/or using a relationship between different variables derived from a real-world situation.

Illustrative examples:

- Comparing the different proportional discounts on a shopping item in two different sales where the discounts are displayed in different ways.
- Understanding and using formulae such as for calculating interest or inflation rates, or one's BMI (Body Mass Index).
- Understanding and applying proportional reasoning to calculate values based on existing percentage or proportions of quantities/ingredients.
- Understanding and applying linear growth in order to predict future growth or decline.

Data and chance

The *Data and chance* content area encompasses topics such as data collection, data displays, charts and graphs, measures of central tendency and variance, alongside understanding appropriate approaches to data collection and sampling. The representation and interpretation of data are key concepts in this category. This content area also includes understanding and knowing about chance and probability. Chance and probability encompass subjective probability, certainty and uncertainty, likelihood and unlikelihood, prediction, and decision making. For example, attaching a numerical value to the likelihood of an instance is a ubiquitous phenomenon no matter whether it has to do with the weather, the stock-market, a medical prognosis or the decision to board a plane.

Illustrative examples:

- Interpreting and identifying particular information on a simple bar graph or pie chart.
- Using an interactive online data tool and chart to interpret and analyse provided data.
- Use and understand averages (mean) to calculate required targets.
- Sort and interpret a set of data to test a number of opinions about the set of data.

Representations

Description[7]

Quantitative and mathematical information in real-world situations and contexts is always represented and embedded in some format or other, whether that be in words and text, or diagrammatically or graphically, or dynamically. Mathematics, per se, does not exist in the real world by itself in its own isolated abstract form, such as 80% x €7.80 such mathematics will be most likely embedded in an advertisement saying “20% discount” and the reader will need to read the information and decide that the solution is to take off 20% of the original price of €7.80. Hence the PIAAC framework needs to elaborate on the different ways that mathematics can be represented in the real world in a numeracy situation.

Mathematical information in a situation may be available or represented in many forms. It may appear as concrete objects to be counted (e.g., people, buildings, cars, etc.) or as pictures of such things. It may be conveyed through symbolic notation (e.g., numerals, letters, and operation or relationship signs). Sometimes, mathematical information will be conveyed by formulae, which are a model of relationships between entities or variables. Mathematical information may be encoded in visual displays such as a diagram or chart; graphs and tables may be used to display aggregate statistical or quantitative information (by displaying objects, counting data, etc.). Similarly, a map of a real entity (e.g., of a city or a project plan) may contain information that can be quantified or mathematised. Last but not least, textual elements may carry much mathematical information or affect the interpretation of mathematical (and statistical) information, as explained further below.

A person may have to extract mathematical information from various types of texts, either in prose or in documents with specific formats (such as in tax forms). Two different kinds of text may be encountered in numeracy tasks. The first involves mathematical information represented in textual form, i.e., with words or phrases that carry mathematical meaning. Examples are the use of number words (e.g., “five” instead of “5”), basic mathematical terms (e.g., fraction, multiplication, percent, average, proportion), or more complex phrases (e.g., “crime rate increased by half”) which require interpretation, or coping with double meanings (or with differences in mathematical and everyday meanings of the same terms). The second involves cases where mathematical information is expressed in regular notations or symbols (e.g., numbers, plus or minus signs, symbols for units of measure, etc.), but is surrounded by text that despite its non-mathematical nature also has to be interpreted in order to provide additional information and context. An example is a bank deposit form or interactive device (e.g., on a mobile device or an *automated teller machine*, ATM) with some text and instructions in which numbers describing monetary amounts are embedded, or a parking ticket specifying an amount of money that has to be paid by a certain date due to a parking violation, but also explaining penalties and further legal steps that will be enacted if the fine is not paid by a certain date.

With the 21st century digitization of information and processes, the types of representation now explicitly include technology-based displays and visualizations on websites, in infographics, in online calculators, spreadsheets and other software and apps on mobile devices and more.

Four classifications for the representation of real-world numeracy situations are described:

- Text or symbols
- Images of physical objects
- Structured information
- Dynamic applications.

For an individual item in PIAAC numeracy, these four descriptions of different representations are not mutually exclusive and any item may involve one or more of these dimensions.

Text or symbols

The stimulus is primarily based on running text that describes the problem situation and can include symbols and numerical information integrated into the text.

Images of physical objects

The stimulus is primarily based on photos or images of physical objects which depicts the problem situation. The image contains the crucial information to solve the problem (e.g. ruler or measuring instrument/scale, 3D objects). Sometimes some text is added to specify or narrow down the problem situation.

Structured information

The stimulus is primarily based on data or information that is represented in tables, graphs/charts, maps, plans, calendars, schedules, timetables, infographics, etc. In most cases, these are computer-generated representations of data, which are becoming more ubiquitous in all news and social media, and in information from government services, financial institutions and utilities. Text will often be used to help specify and describe the information and the problem situation.

Dynamic applications

The stimulus is primarily based on interactive applications, animations, calculation applications (for instance planning and designing software, structured spreadsheets, drawing programs, online applications and calculators such as loan calculators, currency converters, etc.), which are designed to support users to perform calculations or plan or design activities.

This category could also contain: (simulations of) handheld devices and measurement instruments. Sometimes text is used to specify or narrow down the problem situation.

APPENDIX B. PIAAC Numeracy Task Complexity Schema Extract

It should be noted that two of the five factors, both related to the mathematical aspects of the task, had a score range from 1 through to 5, whereas the other three factors only had a range of values of 3 score points. This was based on research work undertaken during the development of the schema that indicated that to reflect the overall item difficulty these two mathematical factors (Complexity of Mathematical Information/data and Type of operation/skill) needed to have a higher overall contribution compared with the other three factors.

The Five Complexity Factors[8]

The literature review suggests that a framework of factors affecting the complexity of numeracy tasks should not only address factors related to the numerical and textual aspects of tasks but should also address other issues. It should treat separately the number of operations and the type of operations from the type of mathematical (or statistical) information to be processed, which may involve numbers explicitly but also other types of mathematical information. In so doing, the desired framework of complexity factors should take into account the broad scope of the definition of numeracy, i.e., reflect the variation within contexts, the range of mathematical ideas/content, the types of possible responses, and the types of representations that cut across adult life contexts.

With the above considerations in mind, five key factors have been identified that are predicted to affect, separately and in interaction, the difficulty level of numeracy tasks to be used in the ALL survey. These five "complexity factors" are outlined in Table 1 and are organized in two sets: two factors that address mainly textual aspects of tasks, and three factors that address the mathematical aspects of tasks. These five factors are listed separately for clarity of presentation, but in actuality are *not* independent of each other and do interact in complex ways. Each factor is examined in some detail below, followed by a later subsection that describes the calculation of an overall complexity level for each item, taking into account all five factors.

Table B1: Complexity Factors—Overview

Aspects	Category	Range
Textual aspects	1. Type of match/problem transparency	Obvious/explicit to embedded/hidden
	2. Plausibility of distractors	No distractors to several distractors
Mathematical aspects	3. Complexity of Mathematical information/data	Concrete/simple to abstract/complex
	4. Type of operation/skill	Simple to complex
	5. Expected number of operations/processes	One to many

Type of Match/Problem Transparency

This is a combination of the factor of Problem Transparency outlined above, and of an IALS factor called Type of Match. Problem Transparency is a function of how well the mathematical information and tasks are specified and includes aspects such as how apparently the procedure is set out, how explicitly the values are stated, etc. Type of Match refers to the process that a respondent has to use to relate the requested action in the question to the information in the task or text, which can range from a simple action of locating or matching to more complex actions that require the respondent to perform a number of searches through the information given. This measure of complexity for a numeracy task incorporates the degree of text embeddedness of the mathematical information.

In easy tasks, the type of information (e.g., numerical values) and the operations needed are apparent and obvious from the way the situation is organized. In more difficult ones, the values must be located or derived from other values; the operations needed may have to be discovered by the performer, depending on his or her interpretation of the context and of the kind of response expected. As well, numeracy situations may involve text to varying degrees, and this text may be of different degrees of importance. There may be a situation where there is little or no text. Some situations may involve pure quantitative information that is to be interpreted or acted upon with virtually no text or linguistic input. In other words, the performer derives all the information needed to respond from the objects present in the situation or from direct numerical displays.

At a higher level, some textual or verbal information may be present alongside the mathematical information. The text can provide background information about the problem situation, or some instructions. For example, a bus schedule, cooking instructions, and a typical school-type word problem all involve some text and some numbers. Still other situations would be heavily text-based or may not involve any numbers or mathematical symbols at all, just plain text. The task will contain mathematical or statistical information that a person needs to understand and, in some cases, act upon, but it will be much less transparent. It may be heavily embedded in dense text or may require using information from a number of sources within or even outside the text/task, or could also mean that outside information (e.g. the understanding and knowledge of a formal formula/process) may be needed to answer the question.

This factor requires that a task will be analyzed in terms of the questions: How difficult is it to identify and decide what action to take?, How many literacy skills are required? and Is all the necessary information there?

Plausibility of distractors

This variable is literacy related, even though it can involve mathematical components. This concerns the extent to which information in the stimulus for the question shares one or more features with the information requested in the question but does not fully satisfy what has been requested. Questions are considered to be easiest when no distractor information is present in the material at all—everything that is needed to answer the question is there, it is explicit with no other distracting information available. Questions tend to become more difficult as the number of distractors increases, as the distractors share more features with the expected response. At higher levels of difficulty, tasks can involve irrelevant information both within the question as well as within the text. In terms of mathematical information, a low level of plausible distractors would mean that no other mathematical information was present apart from that requested, making the numbers or data required to use easy to identify. At a higher level, there may be either some other mathematical information in the task (or its text) that could be a distractor, or the mathematical information given or requested could occur in more than one place. For example, when the numbers required to undertake an arithmetic operation must be extracted from material that contains a range of similar, but irrelevant, information, the task becomes increasingly difficult.

This factor requires that a task will be analyzed in terms of the questions: *How many other pieces of mathematical information are present?*

Complexity of Mathematical Information/data

Some situations present a person with simple mathematical information, such as concrete objects (to be counted), simple whole numbers, or simple shapes or graphs. At lower skill levels, the information will be more familiar, whereas at higher levels, the information may be less familiar. Situations will be more difficult to manage if they involve more abstract or complex information, such as very large or very small numbers, unfamiliar decimals or percents, information about rates, or dense visual information, as in a diagram or complex table.
This factor requires that a task will be analyzed in terms of the question: How complex is the mathematical information that needs to be manipulated or managed?

Type of Operation/Skill

Some situations require simple operations, such as addition or subtraction, or simple measurement (e.g., finding the length of a shelf), or recognition of shape. These are usually easier to analyse mathematically than situations that require multiplication or division, and easier than situations that require using exponents. While the difficulty of recognizing and carrying out the operation implied by a situation (be it additive, multiplicative, etc.) has direct bearing on task complexity, there may be exceptions that occur when alternative approaches are obvious. There are some tasks that combine both interpretive and generative skills and may involve a deeper conceptual understanding than merely carrying out a procedure. Other more complex tasks may involve an explanation of one's reasoning. The interpretation of information appearing in graphs, for example, becomes more complex if comparisons, conjecturing, or "reading beyond the information given" is required.

This factor requires that a task will be analyzed in terms of the question: *How complex is the mathematical action that is required?*

Expected Number of Operations/Processes.

Tasks that require acting upon the mathematical information given may call for one application (step) of an operation, or for one action or process (e.g., literal reading of information in a table, or measurement). More complex tasks will demand more than one operation or process, which may be the same or similar to one another, such as the steps involved in multiple passes on the data or text. Still more complex tasks are those that involve the integration of several different operations or processes.

This factor requires that a task will be analyzed in terms of the question: *How many steps and types of steps/processes are required?*

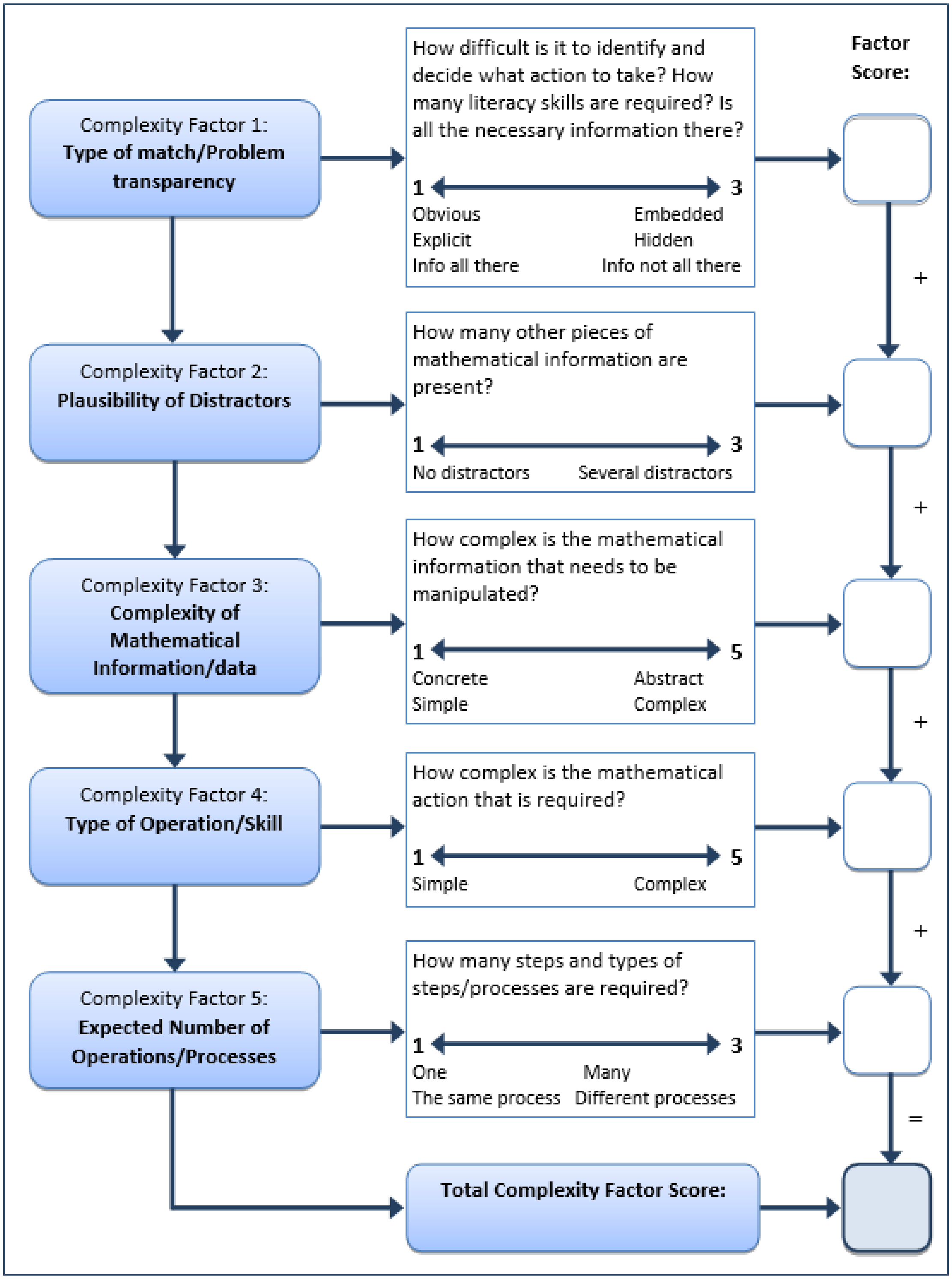
Overall Complexity Level

It is possible to estimate the overall difficulty level of a specific item by first scoring the item on each of the five factors of complexity, according to the levels described in other sections of the schema, and then summing together the scores for each factor. Figure 1 on the following page explains the process. The full paper details and provides scores for each level of the five factors in detail. The total summary score can range between 5 (easiest) and 19 (most difficult).

The estimation process outlined in Figure 1 suggests that each factor has a separate contribution to an item's overall difficulty or complexity. However, it can be hypothesized that as tasks become more complex, actual performance on items may increasingly depend not only on each factor by itself, but also on the interplay or interaction between them. Hence, the computational process suggested in Figure 1 can provide only approximate information about an item's anticipated difficulty level.

Further, the difficulty of a task cannot in some cases be predicted without taking into account characteristics of the person who interacts with the task. The same task may be more difficult for some individuals and less difficult for other individuals, depending on factors such as their familiarity with the context in which a task is situated, knowledge of formal mathematical notations, background world knowledge, as well as general literacy, problem-solving, and reasoning skills. For example, it could be predicted that a task that involves the composition of a fertilizer would be more difficult for an urban apartment dweller than for a rural farmer whereas a task that uses a bus schedule would be more difficult for the farmer. For the above reasons, the prediction of the difficulty of a task in isolation of detailed knowledge about the respondent himself can only be an estimate.

Figure B1. Complexity Flow Chart



Each of these five factors are elaborated in more detail in Tout et al. (2020), and have been utilized in helping to describe the progressions in this paper.

APPENDIX C. Elaborations for each level in Numeracy[9]

Level 1 Proficiency Description – Numeracy

Adults at Level 1 demonstrate number sense involving whole numbers, decimals, and common fractions and percentages. They can access, act on and use mathematical information located in slightly more complex representations set in authentic contexts where the mathematical content is explicit and uses informal mathematical terminology with little text and minimal distracting information. Adults can devise simple strategies using one or two steps for determining the solution.

Adults at this level can:

- interpret simple spatial representations or a scale on a map,
- identify and extract information from a table or graphical representation or complete a simple whole-number bar chart,
- identify the largest value in an unordered list, including comparing the decimal part of the number, and
- interpret and perform basic arithmetic operations, including multiplication and division, with whole numbers, money, and common whole number percentages, such as 25% and 50%.

Cognitive processes	*Content*	*Representation*
At Level 1, adults can • access, act on and use mathematical information located in slightly more complex representations set in authentic contexts where the mathematical content is explicit • use informal mathematical terminology with little text and minimal distractors.	At Level 1, adults can • Identify the largest value in an unordered list, including comparing the decimal part of the number • Perform calculations with small whole numbers and make a calculation with simple percentages (50%) • Interpret a scale in a concrete situation • Locate data directly from a graph and identify a correct percentage in a given situation • Sort events by chronological order and select single data from chosen table • Identify a category in diagram with single data and select multiple data from diagram from a given category	At Level 1, adults can • read and interpret mathematical information that uses simple, familiar and non-formal language and symbols that can incorporate some more complex representations • read and interpret mathematical information set in authentic and mainly familiar and commonplace contexts • read and interpret mathematical content that is explicit and uses informal mathematical terminology with little text and minimal distracting information • respond to closed questions such as identifying or locating straightforward information • respond utilizing some level of interactivity – such as clicking on relevant information.

Level 2 Proficiency Description – Numeracy

At Level 2, adults can access, act on and use mathematical information, and evaluate simple claims, for tasks set in a variety of authentic contexts. They are able to interpret and use information presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, or linear scales) that includes more formal terminology and distracting information. Adults at this level can carry out multistep mathematical processes.

Adults at this level can:

- use dynamic applications to perform simple measurements, and access and sort data given in tables or interactive charts,
- apply simple proportional reasoning or solve problems satisfying up to two conditions,
- formulate processes and expressions to represent situations mathematically, including combining and linking information,
- use mathematical reasoning when reviewing and evaluating the validity of statements,
- estimate or perform calculations involving fractions, decimals, time, measurements, and less common percentages or perform routine algorithms such as that used to generate the mean,
- substitute into and evaluate contexts involving authentic algebraic formulae, and
- identify patterns within two-dimensional geometric representations.

Cognitive processes	*Content*	*Representation*
At Level 2, adults can ● access, act on and use mathematical information, and evaluate simple claims, for tasks set in a variety of authentic contexts. ● interpret and use information presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, or linear scales) that includes more formal terminology and distractors. ● carry out multistep mathematical processes.	At Level 2, adults can ● Make calculations with time, average and various straightforward percentages (15%) ● Provide an appropriate calculation and evaluate statements on numbers in table ● Use tools to convert quantity/unit and perform simple measurements ● Estimate fraction/ratio on a scale ● Select multiple data from diagram and categorize and find right category from table by given data ● Evaluate straightforward statements on data in table	At Level 2, adults can ● read and interpret information presented in slightly more complex forms (e.g., doughnut charts, stacked bar graphs, multiple charts, or linear scales) ● read and interpret some more formal terminology/ language/symbols where some interpretation is required. ● read and interpret information that can be less familiar or common and which is partially embedded but still quite explicit ● read and interpret information that can be located within a number of sources within the text/activity including some distracting information ● interpret representations and visualization and related text where some translation is required ● respond to fairly closed questions that can require a level of interpretation ● respond by utilizing different levels of interactivity – such as locating and clicking on relevant information.

Level 3 Proficiency Description – Numeracy

At Level 3, adults can access, act on, use, reflect on and evaluate authentic mathematical contexts. This requires making judgements about how to use the given information when developing a solution to a problem. The mathematical information may be less explicit, embedded in contexts that are not always commonplace, and use representations and terminology that are more formal and involve greater complexity. Adults at this level can complete tasks where mathematical processes require the application of two or more steps and where multiple conditions need to be satisfied. Tasks may also require the use, integration, or manipulation of multiple data sources in order to undertake the mathematical analyses necessary for the specific task.

Adults at this level can:

- estimate or perform calculations with a wide range of whole numbers, decimals, percentages, fractions, and measurements, including the application of proportional reasoning,
- determine a missing value from a data set given the mean,
- recognize and use patterns (visual and numerical) to estimate values,
- reflect on and use mathematical reasoning when reviewing and evaluating the validity of conclusions drawn from data, including a limited set of related conditions or statements,
- evaluate claims and stated relationships using a variety of data sources,
- recognize a formulation using non-standard notation, and
- use spatial-visualization ability to analyse figures, including moving from three- to two-dimensional representations.

Cognitive processes	*Content*	*Representation*
At Level 3, adults can ● access, act on, use, reflect on and evaluate authentic mathematical contexts: ● Making judgements about how to use the given information when developing a solution to a problem. ● The mathematical information may be less explicit, embedded in contexts that are not always commonplace ● Mathematical processes require the application of two or more steps and where multiple conditions need to be satisfied. ● Tasks may also require the use, integration, or manipulation of multiple data sources in order to undertake the mathematical analyses necessary for the specific task. Adults can: ● Estimate ● Determine missing values ● Recognize patterns ● Reflect and use mathematical reasoning ● Evaluate claims ● Recognize formulation (using non-standard notation) ● Use spatial visualization	At Level 3, adults can ● Perform multistep calculation with percentages, time and multiple variables ● Calculate and reason with maps (2D), sketch plans, and volumes (3D) ● Reason relationally with data and evaluating statements ● Identify a correct formula for a situation (modelling) and use formula to calculate ● Select data from table with multiple criteria or data from more diagrams ● Interpret complex data representations and evaluate more complex statements	At Level 3, adults can ● access, act on, and use information and representations and terminology that are more formal and involve greater mathematical complexity, including algebraic representations and conventions. ● access, act on, and use mathematical information that is less explicit and based in contexts that are less familiar and can require a more formal mathematical interpretation or understanding ● read and interpret information that can be embedded within a number of sources across the text/activity often including some distracting information ● read and interpret information embedded in a more technical, dynamic representation a visualization where interaction and interpretation is required e.g., spreadsheet processes ● bring to the problem related information or knowledge from outside the problem. ● respond to more open/interpretive questions including satisfying different criteria

Level 4 Proficiency Description – Numeracy

At Level 4, adults can use and apply a range of problem-solving strategies to access, analyse, reason, and critically reflect on and evaluate a broad range of mathematical information that is often presented in unfamiliar contexts. Such information may not be presented in an explicit manner. Adults at this level can devise and implement strategies to solve multistep problems. This may involve reasoning about how to integrate concepts from different mathematical content areas or applying more complex and formal mathematical procedures.

Adults at this level can:

- calculate and interpret rates and ratios,
- devise a strategy to compare large data sets,
- read and interpret multivariate data presented in a single graph,
- analyse complex, authentic algebraic formulae to understand relationships between variables,
- reflect and reason mathematically to review and evaluate the validity of statistical or mathematical conclusions, claims or arguments while accommodating relevant conditions, and
- formulate a problem so that the result will be at the required level of specificity to the context of the situation.

Cognitive processes	*Content*	*Representation*
At Level 4 and above, adults can • access, analyse, reason, and critically reflect on and evaluate a broad range of mathematical information that is often presented in unfamiliar contexts. • Such information may not be presented in an explicit manner. • Devise and implement strategies to solve multistep problems. • Reasoning about how to integrate concepts from different mathematical content areas or applying more complex and formal mathematical procedures. Adults can: • Calculate and interpret • Devise strategies • Read and interpret multivariate data • Analyse complex, authentic algebraic formulae • Reflect and reason mathematically • Formulate a problem (problem-posing)	At Level 4 and above, adults can • Interpret and calculate rates and ratios and make calculations with compound units • Analyse and evaluate authentic formula • Model a situation and draws conclusions from it. • Interpret and select multivariate data from multiple sources • Evaluate statements form complex graphical representations and multiple • Reason about appropriate representations of data	At Level 4 and above, adults can • read and interpret, reflect on and evaluate a broad range of mathematical information that is often presented in unfamiliar contexts • read and interpret information that is not accessible or explicit, where the information and tasks required to act on are more highly embedded along with distracting information, and there is a need to interpret or translate the text and task. • read and interpret more complex and formal mathematical terminology and representations including algebraic representations and conventions • use and apply more technical, dynamic representations or processes where some dynamic interaction and interpretation is required e.g., spreadsheet processes • bring to the problem related knowledge and required mathematical reasoning and processes from outside the problem • respond to more complex, open-ended tasks.

Notes

[1] One of the authors of this working paper was involved in the development, writing and implementation of VCAL, and his knowledge of the numeracy frameworks in both VCAL and PIAAC (and also ALL) supports this alignment of the two sets of descriptions of levels of numeracy proficiencies.
[2] Throughout this summary of each of the characteristics of the different dimensions as your progress up the PIAAC numeracy proficiency descriptions, the use of comparative terms such as “slightly more complex” are based on the analysis of all the PIAAC numeracy items which are described generically here, but are elaborated and detailed further in the Appendices and also in the related dimensions. For example, this first aspect of the Cognitive dimension relates closely to the third dimension of Representation. For further quantification and examples of the meaning of such terms please read across the different dimensions and also at the descriptions and examples used in Appendix C.
[3] From *The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (pp. 94–96), by OECD, 2021, OECD Publishing (https://doi.org/10.1787/4bc2342d-en). Copyright 2021 by OECD.
[4] Adapted from *The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (pp. 98–99), by OECD, 2021, OECD Publishing (https://doi.org/10.1787/4bc2342d-en). Copyright 2021 by OECD.
[5] From *The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (pp. 100–103), by OECD, 2021, OECD Publishing (https://doi.org/10.1787/4bc2342d-en). Copyright 2021 by OECD.
[6] From *The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (pp. 104–106), by OECD, 2021, OECD Publishing (https://doi.org/10.1787/4bc2342d-en). Copyright 2021 by OECD.
[7] From *The Assessment Frameworks for Cycle 2 of the Programme for the International Assessment of Adult Competencies* (pp. 109–110), by OECD, 2021, OECD Publishing (https://doi.org/10.1787/4bc2342d-en). Copyright 2021 by OECD.
[8] From *PIAAC Numeracy Task Complexity Schema: Factors That Impact On Item Difficulty* (pp. 10-12), by D. Tout, I. Gal, M. van Groenestijn, M. Manly, and M. J. Schmitt, 2020, Australian Council for Educational Research (https://doi.org/10.37517/978-1-74286-609-3). Copyright 2020 by OECD.
[9] Shaded content is from “Cycle 2: Numeracy: Numeracy Proficiency Levels,” *What PIAAC Measures*, by National Center for Education Statistics, n.d. (https://nces.ed.gov/surveys/piaac/measure.asp#c2_2_3). Copyright 2025 by U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics.

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